Allocative Efficiency and the Productivity Slowdown*

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Abstract

This paper evaluates the contribution of cross-sector allocative efficiency to the productivity slowdown in the US during the 1970s and 2000s. We extend the framework of Oberfield (2013) to derive sufficient statistics for allocative efficiency and decompose aggregate productivity growth in a multi-sector economy with or without input-output linkages. We find approximately two-thirds of the productivity slowdown can be explained by the lack of improvement in allocative efficiency. Furthermore, data shows that increased sector-level volatility is associated with the deterioration of allocative efficiency.

JEL codes: O47; E23.
Keywords: productivity slowdown; allocative efficiency; volatility; adjustment costs.

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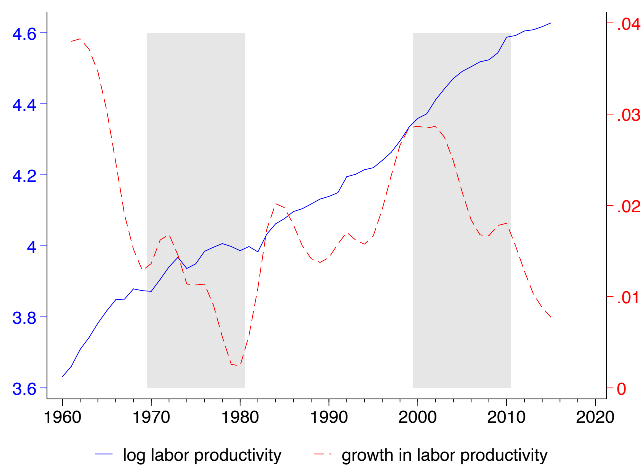
1 Introduction

Growth in US real output per worker decreased significantly in the 1970s and the 2000s (see Figure 1). The slowdown in productivity growth is among the most significant macroeconomic developments in the past few decades and has captured the attention of academic researchers and policymakers. This paper evaluates the role of allocative efficiency across sectors in explaining aggregate productivity dynamics during these periods. We show that allocative efficiency across sectors—or more precisely, the lack of improvement in it—is the common factor behind both episodes of slow productivity growth.

Figure 1: Labor productivity in the US



Notes: This figure plots the logarithms and growth of real output per worker in the United States business sector. The growth rate is computed as the hp-filtered log difference in real output per worker. The slow growth during the 1970s and 2000s is also documented using real output per hour (Figure 1 of Vandenbroucke, 2021) and TFP (Figure 1 of Aum et al., 2018). The gray bars indicate the two productivity slowdown episodes.

We begin by documenting two key facts of allocative efficiency across sectors. To do so, we use a multi-sector model, with and without input-output linkages, and sector-level data from the KLEMS database and the World Input-Output Table to measure allocative efficiency. First, we find that the gradual improvement in allocative efficiency is a significant driver of overall productivity growth in the long run. From 1960 to 2007, allocative efficiency gradually improved, contributing to approximately 20 percent of the aggregate labor productivity growth during this period. Second, the periods of productivity slowdown in the 1970s and

2000s stand out as two exceptions to this long-run trend. Allocative efficiency declined during the 1970s and then stagnated in the 2000s, following two decades of continuous improvement.

From a broader perspective, productivity growth is driven by either (i) enhancements in fundamental productivity (i.e., due to technological advancements) or (ii) improved resource allocation across sectors. If there is a stagnation or decline in allocative efficiency, it can lead to slower productivity growth. While data show that the growth rates of observed labor productivity in the 1970s and 2000s decelerated compared to those of the 1960s and 1990s, our analysis attributes approximately two-thirds of these decelerations to either stagnation or decline in allocative efficiency. Consequently, fundamental productivity contributes to only about a third of the observed slowdown in productivity growth.

Next, we investigate the volatility of sectoral productivity as a potential contributor to the lack of improvement in allocative efficiency during the 1970s and 2000s. Previous theoretical and empirical research has highlighted a mechanism linking higher time-series volatility to a decline in allocative efficiency when (non-convex) adjustment costs are present. The option value associated with making an adjustment results in an inaction region in which firms adopt a wait-and-see approach in adjusting their inputs rather than choosing input quantities that would maximize output in every period. This inaction region broadens with higher volatility as the option value of waiting increases, resulting in a widening gap between the actual allocation of resources and the optimal allocation implied by productivity.

We document a significant variation in volatility over time and across sectors, with higher volatility tending to be associated with a deterioration in allocative efficiency relative to the long-term trend. Through an estimated reduced-form model, we find that this increased volatility plays a substantial role in the observed productivity slowdown. In line with this, sectors subjected to more volatile productivity shocks show slower growth in allocative efficiency. Our analysis further reveals that when sectors experience positive productivity shocks, the amount of resources flowing into those sectors falls short of what the optimal allocation would predict. We also observe that periods characterized by high volatility correspond to an increased dispersion of factor utilization rates across sectors, a pattern possibly driven by the wait-and-see motive.

In summary, our analysis reveals three related findings. First, the gradual improvement in allocative efficiency is a long-run trend, contributing approximately 20 percent of the productivity growth over our sample period. Second, deviations from this trend help give rise to periods of faster- or slower-than-normal productivity growth. And lastly, we emphasize the crucial role of lower volatility in the productivity process for efficient resource allocation, which, in turn, plays a substantial part in driving productivity growth.

Our measure of allocative efficiency follows the tradition of the misallocation literature (Hsieh and Klenow, 2009; Oberfield, 2013; Monge-Naranjo et al., 2019, among others).[1] Our paper also is part of a strand of literature that utilizes sector-level data to examine allocative efficiency between sectors or distortions at the sector level (Basu and Fernald, 2002; Caliendo et al., 2022; Liu, 2019).[2] Our analysis reveals that cross-sector allocative efficiency plays a significant role in explaining aggregate productivity dynamics, in line with findings in Oberfield (2013) and Behrens et al. (2020).

Among the studies exploring the dynamics of productivity in the US during the 1970s and 2000s, a few have examined the role of allocation. Investigating job reallocation across manufacturing industries, Davis and Haltiwanger (2001) propose that oil shocks during the 1970s and 1980s could have created a discrepancy between the actual and desired factor distribution at the industry level. Decker et al. (2020) demonstrate that reallocation across firms significantly decelerated in the 2000s compared to the 1980s and 1990s, suggesting this trend might negatively impact aggregate productivity. Our study diverges from Davis and Haltiwanger (2001) and Decker et al. (2020) in our direct measurement of allocative efficiency, as opposed to analyzing the reallocation rates.

[1]The existing literature, such as studies like Basu and Fernald (2002) and Baqaee and Farhi (2020), adopts different notions of allocative efficiency that diverge from the one used in the misallocation literature. Hence, results derived using these different definitions are not directly comparable.

[2]Note that we use the terms *sector* and *industry* interchangeably here. There also exists a large body of literature utilizing firm- or establishment-level data to study allocative efficiency. An advantage of sector-level data is their broad availability and the extensive time periods they cover. In the specific context of our research, using sector-level data provides us with the opportunity to examine the early slowdown episode (the productivity slowdown of the 1970s).

A majority of papers studying the two slowdown episodes have sought causes that could explain the decline in fundamental productivity. However, as Bloom et al. (2020) note, without accounting for changes in allocative efficiency, the observed productivity dynamics in the raw data differ from those of fundamental productivity. Consequently, the theories developed to explain these dynamics might be flawed. For instance, labor productivity observed in the raw data shows evidence of a gradual decline from 1960 to 2007. In contrast, our result does not show a long-term secular decline in fundamental productivity. Instead, fundamental productivity growth during 1960–2007 can best be characterized as a relatively stable trend marked by prolonged weak growth that began in the 1970s and extended into the 1980s. Recognizing the timing of the slowdown in fundamental productivity could be useful in identifying its underlying causes.

In the same vein, most policy responses to the productivity slowdown primarily target fundamental productivity. Examples include proposals for fiscal and monetary stimuli to boost aggregate demand (Summers, 2018). While these policies may or may not be good ideas, two issues arise from our results. Firstly, the data suggest the decline in fundamental productivity is not as severe as it seems, indicating the need for more moderate subsidies than what the raw data might suggest. Secondly, existing evidence shows that stimulative policies can negatively impact allocative efficiency (Bai et al., 2016) or even promote the survival of zombie firms (Banerjee and Hofmann, 2018), which could counteract any beneficial effects on fundamental productivity.

Previous studies have explored the impact of productivity volatility on allocative efficiency. For instance, Bloom (2009) and Bloom et al. (2018) examine the impact of volatility shocks on allocative efficiency (among other things) in a quantitative model, while Asker et al. (2014) focus on differences in allocative efficiency across countries. Our contribution to the literature lies in identifying volatility as the common factor driving the two episodes of prolonged productivity slowdowns in the US. We do this by providing reduced-form evidence of the mechanisms through an analysis of the time-series properties of allocative efficiency measures and their relationship with the productivity process. The insights of Asker et al. (2014) suggest that a decline in allocative efficiency during periods of heightened volatility may stem from firms' optimization in the presence of adjustment costs. A significant policy implication is that initiatives aimed at reducing the volatility of the productivity process or lowering adjustment costs could be the key to mitigating the issue of prolonged productivity slowdown.

The rest of the paper is organized as follows: Section 2 builds the theoretical framework, and Section 3 discusses the data as well as the mapping between the model and the data. In Section 4, we present the main results. Section 5 concludes.

2 Measuring allocative efficiency

This section presents the theoretical framework. We first characterize optimal allocation across sectors to solve the planner's problem. Then we derive sufficient statistics to measure allocative efficiency. Finally, we decompose aggregate labor productivity growth in the data into two components.

We consider a multi-sector value-added economy. There are N sectors in the economy ($i = \{1, ..., N\}$). In year t, each sector produces a good $Y_{i,t}$ using capital, labor:

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}},$$

where $A_{i,t}$ is the sectoral productivity. There is one final good Y_t, which is produced by aggregating all sectoral goods, such that

$$Y_t = \prod_{i=1}^{N} Y_{i,t}^{\theta_{i,t}},$$

where $\sum_i \theta_{i,t} = 1$. This final good producer is a stand-in for the preference of price-taking consumers, as in Oberfield (2013).

Planner's problem The planner's problem is to allocate aggregate capital K_t and labor L_t into the N sectors to maximize the output of final good Y_t:[3]

$$\max\ Y_t = \prod_{i=1}^{N} Y_{i,t}^{\theta_{i,t}}, s.t.\ Y_{i,t} = A_{i,t} K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}},\ \sum_i K_{i,t} = K_t,\ \sum_i L_{i,t} = L_t. \tag{1}$$

The optimal allocation of capital and labor under problem (1) is such that $K^*_{i,t} = \chi^{k*}_{i,t} K_t$ and $L^*_{i,t} = \chi^{l*}_{i,t} L_t$, where $\chi^{k*}_{i,t} = \frac{\theta_{i,t}\alpha_{i,t}}{\sum_i \theta_{i,t}\alpha_{i,t}}$ and $\chi^{l*}_{i,t} = \frac{\theta_{i,t}(1-\alpha_{i.t})}{\sum_i \theta_{i,t}(1-\alpha_{i,t})}$. The optimal distribution, $\chi^{k*}_{i,t}$ and $\chi^{l*}_{i,t}$, reflect the relative importance of sector i's capital and labor in producing the final good.

Allocative efficiency We define allocative efficiency $\mathbf{E}_t$ as the ratio between output in the data (Y_t) and output under optimal allocation (Y_t^*), such that $\mathbf{E}_t = \frac{Y_t}{Y_t^*}$. We can write $\mathbf{E}_t$ as follows:

$$\mathbf{E}_t = \prod_{i=1}^{N} \left[\left(\frac{\chi^k_{i,t}}{\chi^{k*}_{i,t}} \right)^{\alpha_{i,t}} \left(\frac{\chi^l_{i,t}}{\chi^{l*}_{i,t}} \right)^{1-\alpha_{i,t}} \right]^{\theta_{i,t}}, \tag{2}$$

where $\chi^k_{i,t} = \frac{K_{i,t}}{K_t}$ and $\chi^l_{i,t} = \frac{L_{i,t}}{L_t}$ are sector i's capital and labor as a share of aggregate K_t and L_t in the data, respectively. Intuitively, $(\frac{\chi^k_{i,t}}{\chi^{k*}_{i,t}})^{\alpha_{i,t}}(\frac{\chi^l_{i,t}}{\chi^{l*}_{i,t}})^{1-\alpha_{i,t}}$ measures sector i's allocative efficiency, which represents the deviation of the observed allocation in the data from the optimal allocation. Aggregate allocative efficiency $\mathbf{E}_t$ is the weighted geometric mean of sectoral allocative efficiency with sectoral weights θ_i.[4]

Decomposition of aggregate productivity. According to the definition of $\mathbf{E}_t$, the following equation holds: $Y_t = Y_t^* \mathbf{E_t}$. Dividing both sides by aggregate labor inputs yields the following equation of labor productivity:

$$\mathrm{LP}_t \quad = \quad \mathrm{LP}^*_t \mathbf{E_t}, \tag{3}$$

[3]The optimal allocation problem takes the economy's total capital and labor inputs as exogenously given. Consequently, the measure of allocative efficiency we derive below is a static measure. It abstracts from the dynamic implications of misallocation.

[4]Details of the model can be found in the Appendix Section B.1.

where $\text{LP}_t^* = \frac{Y_t^*}{L_t}$ is the fundamental labor productivity.[5]

Based on this equation, the growth rates of observed labor productivity (grLP_t) and the changes in the growth rates over time (ΔgrLP_t) can be decomposed into a fundamental labor productivity and a allocative efficiency component:

$$\text{grLP}_t = \text{grLP}_t^* + \text{gr}\mathbf{E_t}. \quad (4)$$

$$\Delta\text{grLP}_t = \Delta\text{grLP}_t^* + \Delta\text{gr}\mathbf{E_t}, \quad (5)$$

Both decomposition equations provide insights into observed productivity dynamics. Equation (4) explores the drivers of labor productivity growth within a given period, while Equation (5) investigates why labor productivity growth rates vary across different periods.

Discussions. A few remarks about the framework are in order before moving on to the empirical exercise. Our model abstracts from examining the impact of *within-sector* allocative efficiency on aggregate productivity growth. Conceptually, within-sector efficiency dynamics will be embedded in sector-level productivity, $A_{i,t}$, which we take as given. Therefore, we interpret our results strictly as an analysis of the impact of between-sector allocative efficiency on aggregate productivity dynamics.

The theoretical framework we use in this paper relies on parametrical assumptions about the production function. To ensure robustness, we evaluate two alternative specifications of the production system. First, previous studies suggest that input-output (IO) linkages may affect the measurement of allocative efficiency (Jones, 2013). In Appendix Section A.5, we extend the framework by incorporating input-output linkages. For the period where IO data are available, we find that accounting for IO linkages significantly affects the measured level of allocative efficiency ($\mathbf{E}_t$) but has minimal impact on its growth rates ($\text{gr}\,\mathbf{E}_t$). As highlighted in Equations (4) and (5), it is the growth rates that are crucial for understanding both the

[5] Equation (3) shows that allocative efficiency measures the distance between data (LP_t) and the production possibility frontier (LP_t^*). See discussions in Baqaee and Farhi (2020) for three different notions of allocative efficiency used in the literature.

growth rates and changes in growth rates of observed labor productivity. Thus, our results remain largely unchanged with the incorporation of IO linkages.

Second, we extend the benchmark model to a more flexible CES production system, as detailed in Appendix Section A.6. This extension is motivated by existing studies showing that the measurement of allocative efficiency depends on the elasticity of substitution between sectors and inputs (Epifani and Gancia, 2011; Osotimehin and Popov, 2023). Using the estimated elasticity from our data, we demonstrate that the CES production system yields results similar in magnitude to those of our baseline system. Additionally, we establish the robustness of our findings by testing a wide range of commonly accepted parameter values for elasticity from the literature.

3 Application to the US data

Next, we introduce the datasets used in the empirical analysis and discuss the mapping between data and our model.

3.1 Data description

We use the 2013 version of the KLEMS dataset and restrict our analysis to 28 private sectors in the economy. Table A.1 in the Appendix lists these sectors. Below we list all the variables used in the empirical exercise. We distinguish whether each variable is the nominal value ($) or quantity: (i) sector-level value-added and gross output ($), (ii) sector-level capital and labor compensation, and cost of intermediate goods ($), (iii) sector-level real capital stock and the number of workers (quantity).[6]

[6] The 2013 version of KLMES reports a capital quantity index for each year. An early vintage of EUKLEMS (2009 version) contains real capital stock based on 1995 prices, but the data are only available for a shorter time series. We construct real capital stock for all years using the 1995 real capital stock and the quantity index.

3.2 Mapping between model and data

In this section, we explain the mapping between model and data. Recall that to calculate $\mathbf{E}_t$, we need (i) the allocation of capital, labor and intermediate inputs across sectors in the data $(\chi^k_{i,t}, \chi^l_{i,t}, \chi^y_{i,t}, \gamma_{ij,t})$ and (ii) output elasticities $(\alpha_{i,t},\ \sigma_{ij,t},\ \lambda_{ij,t},\ \theta_{i,t})$, which allows us to solve the optimal allocation.

Cross-sector allocation in the data. First, we calculate the data allocation of capital, labor and intermediate inputs: $\chi^k_{i,t}, \chi^l_{i,t}, \chi^y_{i,t}, \gamma_{ij,t}$. Ideally, we would like to use the quantity of the inputs to calculate the allocation across sectors. We are able to do so for capital and labor, such that $\chi^k_{i,t} = \frac{K_{i,t}}{\sum_i K_{i,t}}$ and $\chi^l_{i,t} = \frac{L_{i,t}}{\sum_i L_{i,t}}$, where $K_{i,t}$ is the real capital stock and $L_{i,t}$ is the number of workers in sector i. Due to the lack of a quantity measure of intermediate inputs, $\gamma_{ij,t}$ and $\chi^y_{i,t}$ are computed using expenditure, such that $\gamma_{ij,t} = \frac{\$d_{ij,t}}{\$Q_{j,t}}$ and $\chi^y_{j,t} = \frac{\$Y_{j,t}}{\$Q_{j,t}}$, where $\$d_{ij,t}$ is sector i's use of sector j good, $\$Q_{j,t}$ is sector j's gross output and $\$Y_{j,t}$ is sector j good used in final good production, all nominal values.[7]

Output elasticities. To solve the planner's optimal allocation problem, we need to know the output elasticities $(\alpha_{i,t},\ \sigma_{ij,t},\ \lambda_{ij,t},\ \theta_{i,t})$.

Among them, $\theta_{i,t}$ comes from the final-good production function. The final-good production function is a stand-in for the preference of households (consumers) in the economy. They represent the demand system in the data, and their value can vary across years, as in Oberfield (2013) and Bils et al. (2020), capturing changes in the demand system over time. We back out $\theta_{i,t}$ from the data using the expenditure share of each sector's output in the final good consumption. In the value-added economy, $\theta_{i,t} = \frac{P^Y_{i,t} Y_{i,t}}{\Sigma_i P^Y_{i,t} Y_{i,t}}$, where $P^Y_{i,t} Y_{i,t}$ represents the value of sector i's value-added output. In the input-output economy, $\theta_{i,t} = \frac{P^Q_{i,t}(Q_{i,t} - \Sigma_j d_{ji,t})}{\Sigma_i P^Q_{i,t}(Q_{i,t} - \Sigma_j d_{ji,t})}$,

[7]Using expenditure as a proxy for quantity will provide an exactly accurate measure of allocative efficiency if the cost of intermediate goods is the same across all sectors. In our model, the measure of allocative efficiency only depends on the distribution of intermediate goods in terms of quantity across sectors. When there is no price variation, the distribution of intermediate goods by expenditure is the same as the distribution by quantity. If there is variation in the cost of intermediate goods across sectors, this approximation will not capture the effect of price dispersion on allocative efficiency.

where $P^Q_{i,t}(Q_{i,t} - \Sigma_j d_{ji,t})$ represents the value of sector i's gross output used for final consumption.

Identification. The key identification challenge arises from the fact that the fundamental technological parameters of the production system – essential for determining the efficient allocation benchmark – are not directly observed in the data. While these parameters can be inferred from firm behavior, such as expenditure shares, the challenge is that expenditure shares may be affected by distortions, potentially obscuring the true underlying technology

To give an example of why these expenditure shares in the data may be distorted: suppose the data is generated by a price-taking representative firm making optimization decisions subject to a set of *wedges* on their capital and labor inputs, represented by $(\tau^k_{i,t}, \tau^l_{i,t})$.[8] The first order conditions lead to the following equation:

$$\frac{R_t K_{i,t}}{w_t L_{i,t}} = \frac{(1-\tau^l_{i,t})\alpha_{i,t}}{(1-\tau^k_{i,t})(1-\alpha_{i,t})},$$

where $R_t K_{i,t}$ and $w_t L_{i,t}$ are the capital and labor expenditure incurred by the firm, respectively. We observe these expenditures in the data, and we can calculate the left-hand side of the above equation. But without any assumptions, $\alpha_{i,t}$ cannot be identified separately from $\tau^k_{i,t}$ and $\tau^l_{i,t}$ on the right-hand side.

Returning to our problem, in an extreme case, if we have unrestricted freedom in selecting these parameters over time, it would become impossible to use data on output and input to infer dispersion in marginal products. Therefore, our approach – like those commonly adopted in the literature – relies on certain restricted but reasonable assumptions. To this end, our analysis examines a wide range of plausible identification assumptions, which can be grouped into two broad categories based on whether the production technology is assumed to change over time.

[8] The wedges may reflect statutory provisions such as tax code and regulations, discretionary provisions such as preferential treatment by the government, and market frictions and imperfections (Restuccia and Rogerson, 2017).

The first class of specifications assumes that technology evolves over time rather than remaining fixed. This perspective is also adopted by Oberfield (2013), whose approach we use to estimate the technology parameters. The underlying assumption is that the expenditure shares observed in the data reflect both slow-moving technological changes and year-to-year fluctuations caused by labor or capital distortions. Crucially, over the long run, sectoral distortions are assumed to be unbiased toward either capital or labor. However, in any given year, distortions may disproportionately impact one factor due to the nature of the shocks. This assumption implies that within a sufficiently long rolling window, the average values of capital and labor wedges converge, allowing us to infer technology parameters from the average expenditure shares observed within the window.

In practice, we perform this analysis by computing factor expenditure shares and averaging them within a rolling window centered on the current year. We test rolling windows of 3, 5, 7, and 9 years and find minimal differences in the results. We also consider the possibility that distortions may be systematically biased against either capital or labor. For instance, economic fundamentals could lead to persistently higher distortions for capital relative to labor, or vice versa. To address this, we explore two alternative scenarios: one where the capital wedge is significantly larger than the labor wedge, and another where the reverse is true. Even under extreme assumptions about the degree of bias, the results remain closely aligned with the baseline findings.

The alternative perspective, upon which our second class of specifications is based, assumes that technology is hard-wired and does not change over time. This approach parallels the assumption in Hsieh and Klenow (2009), where the authors posit that China, India, and the United States share the same production technology at the sector level. Since the U.S. experiences the least distortions among the three countries, its expenditure share data is used to infer technology parameters, which are then applied to the other two countries. Similarly, our analysis uses data from the later years in the sample, which existing studies suggest are less distorted than earlier years, to infer technology parameters and apply them across the entire sample period.[9]

[9]We used two years from our sample to estimate the technological parameters: (i) 2010, the final year of

One might argue that while distortions may be lower in the later years, they are not entirely absent, meaning the estimated technology parameters might still include a distortion component. While this is true, it is important to emphasize that the focus of our paper is on changes in allocative efficiency over time, rather than its absolute levels. Even if the technology inferred from the later years reflects true technology plus some residual distortion, deviations from this base year still capture the impact of changing distortions on allocative efficiency. Although these measures might not reflect the exact level of allocative efficiency, they remain informative statistics, as our primary focus is on changes in allocative efficiency over time.

To summarize, our analysis encompasses a wide range of plausible identification assumptions. In the main text, we closely follow the approach in Oberfield (2013), assuming that technology evolves over time and applying a 3-year rolling window to estimate these parameters. In Appendix Section A.3, we demonstrate that our findings are robust across all other specifications discussed above. Thus, the baseline result should be interpreted alongside the alternative specifications, which collectively provide a comprehensive and robust view of our main findings.

4 Results

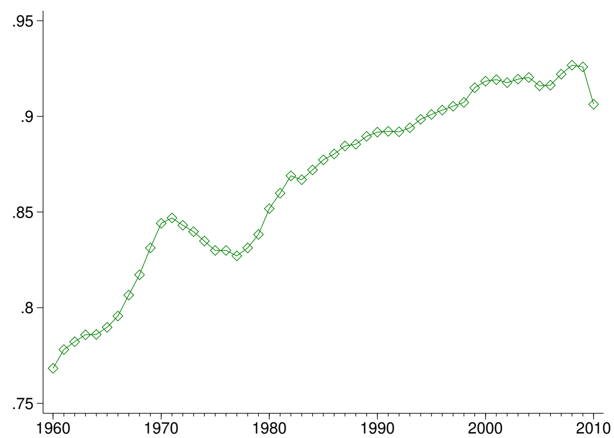
This section presents the main results of the paper. We first examine the evolution of allocative efficiency and establish its role in explaining productivity growth, both in the long run and during the slowdown. Then we provide evidence of the relationship between volatility and slow growth in allocative efficiency.

our sample, and (ii) 2005, to avoid the influence of the Great Recession. Results from these two years show minimal differences.

4.1 Contribution of allocative efficiency to productivity slowdown

Figure 2 illustrates the dynamics of allocative efficiency over time. Between 1960 and 2010, allocative efficiency showed a gradual upward trend, approaching the optimal value of 1. However, this progress was disrupted by two notable exceptions: (i) a decline in the 1970s and (ii) a plateau in the 2000s following two decades of continuous improvement.

Figure 2: Evolution of allocative efficiency over time



Notes: This figure displays the cross-sector allocative efficiency ($\mathbf{E}_t$), measured using the KLEMS dataset, for the baseline economy during 1960-2010.

We begin by analyzing how allocative efficiency influences productivity growth over our sample period. Between 1960 and 2010, measured allocative efficiency increased by 18 percent (from 0.77 to 0.91), while observed labor productivity grew by 89 percent. The decomposition equation (4) shows that a 1 percent increase in allocative efficiency directly translates into a 1 percent increase in labor productivity. These figures suggest that approximately 21 percent (18/89) of the observed productivity growth can be attributed to improvements in allocative efficiency.

Allocative efficiency contributes not only significantly to long-run productivity growth but also to the productivity slowdowns of the 1970s and 2000s. Here, a productivity *slowdown* refers to the periods when the growth rates of productivity were considerably slower than usual. In other words, a slowdown is not about the productivity growth rates per se but rather about how these rates vary over time – faster in some periods and slower in others.

To illustrate the extent of these productivity slowdowns, we compute the decadal growth rates of labor productivity, presented in Column (1) of Table 1. Column (4) of the same table, which shows the first differences of these decadal growth rates, highlights the changes in growth rates compared to the preceding decade. The 1970s and 2000s are the only decades with negative values in Column (4), indicating that productivity growth rates during these periods were lower than in the prior decades. Specifically, observed labor productivity growth rates were 12 percentage points and 3 percentage points lower in the 1970s and 2000s, respectively, compared to the preceding decades.[10]

Table 1: Slowdown in productivity growth and the role of allocative efficiency

	Panel (a) Growth rates by periods (long log-difference)			**Panel (b)** Changes in growth rates from preceding period		
	(1)	(2)	(3)	(4)	(5)	(6)
	labor productivity			labor productivity		
Period	data	"fundamental"	$\mathbf{E}_t$	data	"fundamental"	$\mathbf{E}_t$
1960–69	0.24	0.16	0.08	–	–	–
1970–79	0.13	0.13	-0.01	**-0.12**	**-0.03**	**-0.08**
1980–89	0.15	0.10	0.04	0.02	-0.03	0.05
1990–99	0.19	0.16	0.03	0.05	0.06	-0.02
2000–07	0.16	0.16	0.01	**-0.03**	**-0.01**	**-0.02**

Notes: Columns (1)–(3) of Panel (a) present the growth rates of LP_t (labor productivity, data), LP_t^* (labor productivity, fundamental), and $\mathbf{E}_t$ (allocative efficiency). These growth rates are calculated as long differences in labor productivity between the start and end of each period and are adjusted to reflect growth rates over consistent ten-year windows. Panel (a) provides the decomposition of the decadal labor productivity growth rate based on Equation (4), where the sum of Columns (2) and (3) equals the value in Column (1), before rounding. In Panel (b), Columns (4)–(6) present the changes in the growth rates from Panel (a) relative to the preceding decades. Panel (b) uses Equation (5) to decompose these changes in labor productivity growth rates. Accordingly, the sum of Columns (5) and (6) corresponds to the value in Column (4) before rounding.

Next, we examine how much of the productivity growth slowdowns can be attributed to allocative efficiency. Using Equation (4), we decompose the decadal growth rates in observed productivity into growth rates of fundamental productivity (Column 2) and allocative efficiency (Column 3). Columns (5) and (6) calculate the first differences of these growth

[10]In the baseline exercise, we conducted our analysis in decades, focusing on the 1970s and post-2000s as periods of slowdown. However, the timing of the slowdown may not align precisely with these decade-long intervals. To address this, Appendix Section A.7 provides a more detailed analysis of the start and end dates of the slowdown episodes, based on identified breaks in labor productivity growth trends..

rates, revealing that both allocative efficiency and fundamental productivity slowed during the 1970s and 2000s, as reflected in the negative values for these decades.

This outcome highlights that the productivity slowdown was driven by both slower growth in allocative efficiency and fundamental productivity. Specifically, Column 6 shows that allocative efficiency growth slowed by 8 pps and 2 pps during the 1970s and 2000s, respectively, accounting for approximately two-thirds (8/12 and 2/3) of the observed labor productivity slowdown. The remaining one-third is attributable to the slowdown in fundamental productivity growth.

Table 1 also provides valuable insights into the long-term trends in productivity. There has been extensive discussion about a secular decline in productivity growth over time. Column (1) of the table, which presents observed labor productivity growth rates by decade, indeed shows a deceleration in productivity growth. This deceleration becomes particularly evident when comparing the slower growth rates of later decades to the exceptionally rapid 24% growth during the 1960s. However, a closer look at Column (3) suggests that the 1960s also experienced significant growth in allocative efficiency. This observation highlights the importance of distinguishing between the impacts of allocative efficiency and fundamental productivity when assessing the magnitude and causes of secular stagnation. After removing the impact of allocative efficiency, there is no evidence of a secular deceleration in fundamental productivity growth (Column 2). Instead, fundamental productivity growth follows a relatively stable trend, characterized by prolonged weak growth that began in the 1970s and extended into the 1980s. These findings have important implications for the ongoing discussion around secular stagnation.

Robustness. We establish the robustness of these results through several exercises. The first set of exercises examines different identification assumptions, as detailed in Appendix Section A.3 and discussed at the end of the previous section. Next, we address potential measurement issues of the KLEMS data in Appendix Section A.4. In this exercise, we use alternative measures for capital and labor inputs, accounting for the possibility that sectors may have different compositions of various types of capital and labor. Since the KLEMS data assumes zero average profits at the sector level, we also consider scenarios where profit shares are non-zero. Finally, we address several limitations of the data by conducting additional robustness checks. Specifically, we use a different version of the KLEMS dataset to explore post-2010 dynamics (Appendix Section A.9) and analyze the productivity slowdown in the manufacturing sector at a more granular level using data from the NBER-CES database (Appendix Section A.10).[11]

4.2 Decomposing allocative efficiency

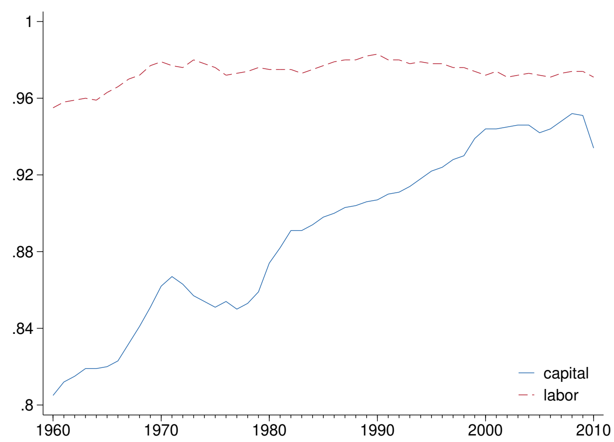
Next, we carry out two decomposition exercises to further investigate the dynamics of aggregate allocative efficiency. First, we break down the aggregate allocative efficiency into the respective allocative efficiencies of capital and labor. Following this, we dissect the aggregate measure into sectoral allocative efficiency.

Capital and labor. We decompose $\mathbf{E_t} = E_t^k \cdot E_t^l$ to identify whether capital or labor is the main driver. We define the measures of capital and labor allocative efficiency in the value-added economy as $E_t^k = \prod_{i=1}^N (\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}})^{\alpha_{i,t}\theta_{i,t}}$ and $E_t^l = \prod_{i=1}^N (\frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}})^{(1-\alpha_{i,t})\theta_{i,t}}$, and for the input-output economy, the measures are similarly defined.[12]

Figure 3 reveals that capital allocation is the more important driver across the sample period. In contrast, labor allocation is more efficient than capital allocation in almost all years, but it did not become more efficient over time. Indeed, labor allocation seems so efficient that it has little room for improvement and thus cannot possibly be a large driver. As with the long-run dynamics, capital allocation plays a more significant role during the two slowdown episodes.[13]

[11] The main variables in the KLEMS dataset, such as capital stock and output, are constructed from national accounts. Due to its relatively aggregate nature, KLEMS likely suffers less from the measurement error issues commonly found in firm-level data used in the literature (see discussion in Bils et al., 2021). However, the KLEMS dataset lacks certain granularity. To address this, we use the NBER-CES dataset, which provides industry-level data covering 364 manufacturing industries, to show that our findings are also

Figure 3: Capital and labor allocation, E_t^k and E_t^l



Notes: This figure plots the evolution of capital (E_t^k) and labor allocation (E_t^l) over time, defined by $E_t^k = \prod_{i=1}^N (\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}})^{\alpha_{i,t}\theta_{i,t}}$ and $E_t^l = \prod_{i=1}^N (\frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}})^{(1-\alpha_{i,t})\theta_{i,t}}$.

Sectoral allocative efficiency. To discern whether a particular sector drives the overall trends, we decompose the aggregate measure into sectoral allocative efficiency, represented as $\mathbf{E}_t = \prod_{i=1}^N E_{i,t}^{\theta_{i,t}}$, where $E_{i,t} = (\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}})^{\alpha_{i,t}}(\frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}})^{1-\alpha_{i,t}}$ is the allocative efficiency of sector i.[14]

Figure 4, Panel (a) presents the distribution of $E_{i,t}^{\theta_{i,t}}$, where different shades of colors correspond to different percentiles of the distribution. According to its definition, $E_{i,t} = 1$ means that sector i is at the optimal level. Therefore, an optimal allocation would see the cross-sectional distribution of $E_{i,t}$ collapse to a single point ($E_{i,t} = 1$ for all i), and a narrower distribution indicates a more efficient allocation. Notably, the distribution significantly narrows between 1960–1970 and 1980–2000 but widens in the 1970s and stabilizes post-2000, mirroring aggregate allocative efficiency trends. Furthermore, the changes in $E_{i,t}$ observed at different percentiles indicates that these aggregate dynamics are not driven by a single sector.

evident at a more detailed level.

[12] $E^{k,t} = \prod_{i=1}^N(((\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}})^{\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}})^{\sum_n \theta_{n,t}C_{ni,t}}$ and $E^{l,t} = \prod_{i=1}^N(((\frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}})^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}})^{\sum_n \theta_{n,t}C_{ni,t}}$.

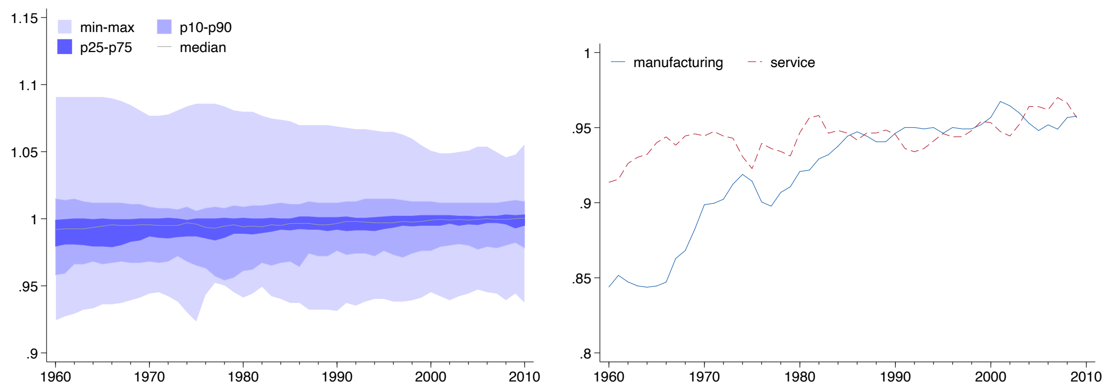
[13] However, given the data do not differentiate between capital returns and profits, caution is necessary when interpreting these results, particularly for policy implications.

[14] We discuss results for the value-added economy in the main text as the input-output model yields similar results.

We also examine the allocative efficiency of the manufacturing and service sectors, represented in Figure 4, Panel (b).[15] Both sectors experience a decline in allocative efficiency in the 1970s, with the service sector recovering slightly earlier than manufacturing. Conversely, the 2000s see a more concentrated drop in allocative efficiency of the manufacturing sectors.

Figure 4: Sector-level allocative efficiency

(a) Full distribution **(b)** Manufacturing and service sector



Notes: Panel (a) plots the distribution of $E_{i,t}$ in a model without input-output linkages. The different shades of color represent different percentiles of the $E_{i,t}$ distribution in year t. The grey line represents the median value of $E_{i,t}$. Panel (b) plots the allocative efficiency for the manufacturing and service sectors in both models. The classification of manufacturing and service sectors can be found in footnote 15.

To summarize, our analysis underscores the significant role of allocative efficiency in explaining the deceleration of aggregate productivity growth during the 1970s and 2000s. Moreover, we find that capital is the primary driver of change in allocative efficiency. The sluggish growth in allocative efficiency during productivity slowdown episodes can be attributed to sectors collectively moving away from the optimum. In Appendix Section A.2, we conduct an in-depth analysis of the long-run changes in allocative efficiency. The remainder of Section 4 will delve into an investigation of a potential driver behind the patterns of allocative efficiency during the two slowdown episodes.

[15] The manufacturing and service sectors are defined as $E_t^m = \prod_{i \in manu} E_{i,t}^{\theta_{i,t}}$ and $E_t^s = \prod_{i \in serv} E_{i,t}^{\theta_{i,t}}$, respectively. See Table A.1 for sector classification details.

4.3 Volatility and allocation during the slowdown episodes

We explore the role of volatility of sector-level productivity in explaining the cross-sector allocative efficiency during the 1970s and 2000s. First, using a variety of measures, we document that the 1970s and 2000s experienced an increase in sector-level volatility. Next, we exploit the variations in volatility over time and across sectors to provide evidence linking the increase in volatility to the deterioration in allocative efficiency.

Theoretical underpinnings. Previous studies have shown a mechanism linking higher time-series volatility with a decline in allocative efficiency (Bloom et al., 2018 and Asker et al., 2014). Specifically, in the presence of non-convex adjustment costs, there is an inaction region in which firms adopt a wait-and-see approach in adjusting their inputs rather than choosing input quantities that would maximize output in each period. With higher volatility, this inaction region expands as the option value of waiting increases, resulting in a wider gap between firms' chosen allocation and the optimal allocation suggested by productivity; hence leading to lower allocative efficiency.[16]

Although this mechanism is described at the firm level, it can also emerge at the sector level through the aggregation of firm-level behaviors.[17] To understand this aggregation result, consider an environment where a sector consists of many firms, with each firm's productivity represented as the product of a sector-level component and a firm-specific component. An increase in sectoral productivity raises the productivity of all firms in that sector. Similarly,

[16] The literature identifies two main effects of increased volatility in the presence of adjustment costs. The first, known as the *option value effect*, enlarges the inaction regions, creating a greater disparity between actual and optimal allocations. The second termed the *volatility effect*, triggers a medium-term surge in aggregate hiring and investment. Research indicates that, in the medium term, the volatility effect tends to dominate, leading to an overshoot in aggregate labor and investment. However, this overshoot does not extend to allocative efficiency: as shown in Figures 6 and 7 of Bloom et al. (2018), labor misallocation worsens immediately after a shock and gradually returns to pre-shock levels without exceeding them. In contrast, aggregate labor and investment exceed their pre-shock levels 3 to 6 quarters following the shock.

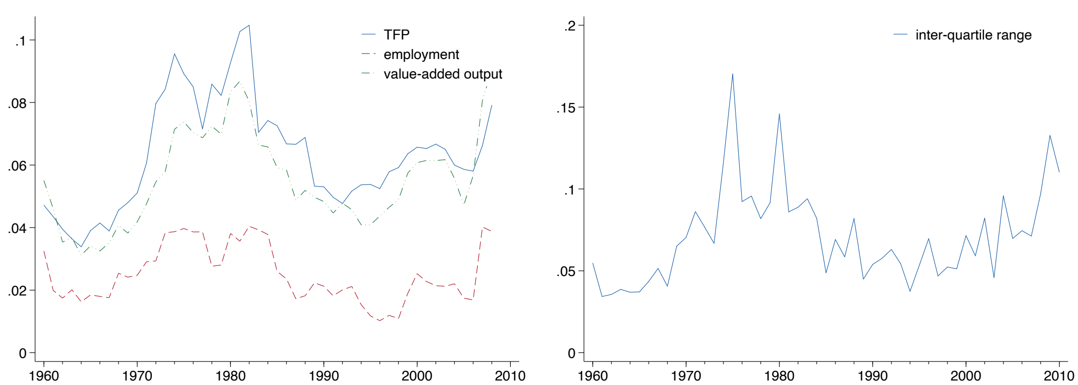
[17] Existing studies in the literature have examined cases where adjustment costs and input choices occur at a finer level than the observed data. For instance, in Bloom (2009), while the data is at the firm level, input choices in the model are made at the establishment level, with each firm comprising multiple establishments. Bloom finds that the qualitative properties of the underlying mechanism are preserved at the (higher) firm level, even though aggregating outcomes to the firm level smooths out sharper adjustments in capital and labor observed at the establishment level.

a rise in the volatility of sectoral productivity increases productivity volatility across all firms. Now consider a situation where sector i's productivity level increases relative to other sectors. In the presence of adjustment costs, firms of sectors i that are within the inaction region do not increase their inputs, while firms outside of the inaction regions adjust to the new optimal levels of capital and labor. Aggregating the behavior of all firms implies that sector i as a whole will employ less capital and labor than the increased productivity level would optimally imply, leaving the rest of the economy with more capital and labor than optimal and lowering the allocative efficiency across sectors. Higher sectoral volatility would exacerbate this mechanism as more firms fall into the inaction region.

Productivity slowdown episodes are accompanied by high volatility. As a first pass of the data, we calculate the sectoral growth volatility in employment, real value-added output, and TFP, using the standard deviation of their respective annual growth rates over a rolling 5-year window. As shown in Panel (a) of Figure 5, volatility begins to rise at the onset of the 1970s, remaining high throughout the decade. At the highest point, standard deviations were nearly double those of the 1960s. Volatility then gradually decreases between the 1980s and the early 2000s, after which it starts to ascend again.

Figure 5: Sector-level shocks were more volatile during the 1970s and 2000s

(a) S.D. in sectoral growth rates **(b)** Dispersion in TFP shocks



Notes: Panel (a) plots the cross-sectional s.d. of sectoral growth rates in employment, real value-added output, and TFP. Panel (b) plots the cross-sectional dispersion (inter-quartile range) in TFP shocks, computed as the residual terms from regression $\log A_{i,t} = \rho \log A_{i,t-1} + \mu_t + \chi_i + \epsilon_{i,t}$.

Next, we construct a measure of volatility based on dispersion in sector-level TFP. Following Bloom et al. (2018), we first calculate TFP shocks as the residual ($\varepsilon_{i,t}$) from a regression equation for sector-level log TFP, $\log A_{i,t} = \rho \log A_{i,t-1} + \mu_t + \chi_i + \varepsilon_{i,t}$, and use the cross-sectional dispersion of $\varepsilon_{i,t}$ to measure the volatility of sectoral TFP processes.[18] Panel (b) of Figure 5 shows the inter-quartile range of $\varepsilon_{i,t}$ within each year. The 1970s and 2000 are again marked by greater volatility.[19]

Relationship between volatility and allocative efficiency. Our next step is to systematically evaluate the relationship between volatility and allocative efficiency by exploiting variations in volatility over time. The results are shown in Table 2. Columns (1)–(3) regress aggregate allocative efficiency $\log(\mathbf{E}_t)$ on the volatility of TFP shocks in year t, $t-1$, and $t-2$, while controlling for $\log(\mathbf{E}_{t-1})$. The inclusion of $t-1$'s and $t-2$'s measure of volatility is motivated by the insight that the history of TFP shocks might have a long-lasting impact on allocation in the presence of adjustment costs. The inclusion of $\log(\mathbf{E}_{t-1})$ aims at controlling for the long-run trend.

The results confirm that higher volatility in year t is associated with a significantly less efficient allocation, with an estimated coefficient of -0.072 to -0.087. In Columns (4)–(6), we find very similar results when using the changes in allocative efficiency as the dependent variables. Again, the estimated correlations range from -0.081 to -0.097 and remain highly significant.[20]

[18] This regression controls for the sector- and year-fixed effects, which removes permanent differences in sectoral TFP levels and removes average growth rate differences across years.

[19] Appendix Figure A.11 plots an alternative measure of sector-level volatility, in which we first compute the over-time dispersion of $\varepsilon_{i,t}$ within a rolling window for each sector and then calculate the median value of these dispersions among all sectors. We find a similar dynamic as Figure 5, Panel (b), particularly in the rising volatility during the 1970s and 2000s. Our paper does not explore the factors underlying the rise in volatility. However, we note that existing studies have examined the policy origins of increased economic volatility in the US. For example, Baker et al. (2016) measured policy uncertainty and found a positive association with higher economic volatility in the US.

[20] We note two things about the estimation results. First, R^2 is high in Columns (1)–(3) due to controlling for the lagged value of the dependent variable (allocative efficiency in year $t-1$). Second, the estimated coefficients for TFP dispersion are similar in Columns (1)–(3) and Columns (4)–(6) despite the use of different dependent variables. This similarity exists because the estimated coefficients for the lagged dependent variable in Columns (1)–(3) are close to 1, effectively making the regression models similar in these columns and resulting in similar estimates.

Table 2: Relationship between volatility and allocative efficiency

	(1)	(2)	(3)	(4)	(5)	(6)
Dispersion of TFP shocks in year t	-0.072**	-0.087**	-0.080**	-0.081**	-0.097**	-0.095**
	(0.031)	(0.037)	(0.037)	(0.032)	(0.038)	(0.039)
Dispersion of TFP shocks in year $t-1$		0.003	0.009		0.005	0.007
		(0.043)	(0.046)		(0.041)	(0.046)
Dispersion of TFP shocks in year $t-2$			-0.006			0.002
			(0.027)			(0.030)
Allocative efficiency in year $t-1$	0.973***	0.981***	0.976***			
	(0.013)	(0.012)	(0.012)			
Dependent variables	$\log(\mathbf{E}_t)$	$\log(\mathbf{E}_t)$	$\log(\mathbf{E}_t)$	$\Delta\log(\mathbf{E}_t)$	$\Delta\log(\mathbf{E}_t)$	$\Delta\log(\mathbf{E}_t)$
N	63	62	61	63	62	61
R^2	0.993	0.993	0.993	0.093	0.137	0.128
Observed slowdown in 1970s	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12
Predicted slowdown in 1970s	-0.10	-0.10	-0.10	-0.07	-0.08	-0.08
Observed slowdown in 2000s	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
Predicted slowdown in 2000s	-0.02	-0.03	-0.03	-0.03	-0.03	-0.03

Notes: The top half of the table presents the regression results. Columns (1)–(3) regress logarithms of $\mathbf{E_t}$ on the cross-sectional dispersion in TFP shocks from years $t, t-1, t-2$ while also controlling for the logarithms of $\mathbf{E_{t-1}}$. Columns (4)–(6) regress the log difference in $\mathbf{E_t}$ on the cross-sectional dispersion in TFP shocks of years $t, t-1, t-2$. Robust standard errors are reported and in parentheses. The bottom half of the table presents the predicted productivity slowdown using the estimated models. The four rows correspond to the same two productivity slowdown periods in Table 1: 1970–79 (1970s) and 2000–07 (2000s). The predicted growth rates are computed as $\Delta\log\widehat{\mathrm{LP}}_t = \Delta\log\mathrm{LP}_t^* + \Delta\log\widehat{\mathbf{E}}_t$, where $\Delta\log\mathrm{LP}_t^*$ are taken from previous estimates in Table 1. The observed and actual growth rates are calculated as the long difference between the beginning and end of each period. They are adjusted to reflect the growth rate over periods of the same length (ten-year window). The slowdown in productivity growth (observed and predicted) is calculated as the changes in growth rates from the previous periods.

These regression results indicate that the impact of increased volatility on productivity growth can be economically significant. For instance, compared to the 1960s, the inter-quartile range of TFP shocks increased by 0.052 in the 1970s. This increase, according to the estimates in Table 2, translates into a decline in the left-hand-side variable, $\log\mathbf{E}_t$, by a size ranging from 0.004 to 0.005. Furthermore, in the data, the average annual growth rate of labor productivity was approximately 0.015 during the 1970s. This means that if the volatility of TFP shocks in the 1970s had stayed the same as the previous decade, the annual productivity growth rate would have been approximately 30 percent higher.

Next, we quantify to what extent volatility slows down productivity growth. To answer this question, we first obtain the predicted value for $\widehat{\mathbf{E}}_t$ using estimates from Table 2, then calculate predicted growth rates using the following equation: $\Delta \log \widehat{\mathrm{LP}}_t = \Delta \log \mathrm{LP}^*_t + \Delta \log \widehat{\mathbf{E}}_t$.[21] As shown at the bottom of Table 2, model-predicted slowdown during the 1970s ranges somewhere between 8 pp and 10 pp when we apply the full set of regressors (Columns 3 and 6), accounting for more than two-thirds of the observed 12 pp slowdown in the data. Further, the model-predicted slowdown during the 2000s is 3 pp, accounting for the entire observed slowdown in the data. According to Table 1, $\Delta \log \mathrm{LP}^*_t$ slows down by 3 pp during the 1970s and by 1 pp during the 2000s, indicating that the remaining 5–7 pp and 2 pp, respectively, are predicted by the increase in volatility. Based on these findings, volatility plays an even greater role than fundamentals in driving the productivity slowdown.

Evidence at the sector level. As of now, we have established the correlation between volatility and aggregate allocation efficiency. Similarly, allocative efficiency should also decline in sectors facing high volatility. We next provide evidence linking these two phenomena by exploiting differences in volatility across sectors.

Before we delve into the results, it would be helpful to clarify how we measure the changes in sectoral allocative efficiency. It is important to remember that $E_{i,t}$ could be greater or less than 1 with $E_{i,t} = 1$ indicating that resources allocated to this sector are at the optimal level. Moreover, a decrease in $E_{i,t}$ does not indicate worsening allocation. For instance, a shift from $E_{i,t} = 1.2$ to $E_{i,t} = 0.9$ actually indicates an improvement in allocation efficiency, as the gap between $E_{i,t}$ and 1, which represents how distant the sector is from optimum, decreases from 0.2 to 0.1. As such, the appropriate measure for sectoral allocation efficiency is the absolute difference between $E_{i,t}$ and 1, which in logarithmic terms can be expressed as $|\log E_{i,t}|$. Accordingly, the change in sectoral allocative efficiency is captured by $|\log E_{i,t}| - |\log E_{i,t-\Delta t}|$, wherein a positive value indicates a deterioration in allocation over the period $[t - \Delta t, t]$.

[21] For the regressions in Columns (4)–(6), we first obtain the predicted growth rates $\Delta \log \widehat{\mathbf{E}}_t$, and using the first year of $\mathbf{E_t}$ as a starting point, we compute $\widehat{\mathbf{E}}_t$ recursively for each year. The growth rate under optimal allocation, $\Delta \log \mathrm{LP}^*_t$, is taken from Table 1.

Table 3: Sector-level relationship between volatility and allocative efficiency

Dependent variable $\lvert \log E_{i,t} \rvert - \lvert \log E_{i,t-\Delta t} \rvert$		$\Delta t = 2$			$\Delta t = 4$	
Dispersion of TFP shock in $[t - \Delta t, t]$	0.141**	0.168*	0.154*	0.235**	0.253**	0.224*
	(0.0713)	(0.0866)	(0.0925)	(0.101)	(0.124)	(0.134)
Sector FEs	N	Y	Y	N	Y	Y
Year FEs	N	N	Y	N	N	Y
N	1593	1593	1593	1593	1593	1593
R^2	0.021	0.060	0.104	0.029	0.081	0.127
		$\Delta t = 6$			$\Delta t = 8$	
Dispersion of TFP shock in $[t - \Delta t, t]$	0.317***	0.437***	0.396***	0.541***	0.722***	0.671***
	(0.0895)	(0.120)	(0.134)	(0.123)	(0.173)	(0.191)
Sector FEs	N	Y	Y	N	Y	Y
Year FEs	N	N	Y	N	N	Y
N	1593	1593	1593	1593	1593	1593
R^2	0.056	0.139	0.188	0.070	0.181	0.231

Notes: This table regresses changes in allocative efficiency over $[t - \Delta t, t]$ on growth volatility—measured by the dispersion of TFP shocks—at the sector level. The regressions with or without sector- and year-fixed effects are presented in different columns. In addition, we also show the regression results with rolling windows of different lengths, where $\Delta t = 2$, $\Delta t = 4$, $\Delta t = 6$, and $\Delta t = 8$ correspond to 3-year, 5-year, 7-year and 9-year rolling windows, respectively. Robust standard errors are reported in parentheses.

Table 3 presents the correlation estimates between each sector's volatility, measured by the dispersion in productivity shock $\varepsilon_{i,t}$ over a rolling window from $t - \Delta t$ to t, and the associated changes in allocative efficiency within that same window. With a three-year rolling window ($\Delta t = 2$), the estimated coefficient on the dispersion of TFP shocks stands at 0.14, and this value increases somewhat upon controlling for one or both sets of sector- and year-fixed effects. For longer rolling windows ($\Delta t = 2, 4, 6, 8$), a consistently significant relationship is found between heightened volatility and decreased allocative efficiency. Taken together, results in this table suggest that volatility increases are linked to notable declines in allocative efficiency at the sector level.

The mechanism highlighted here also has predictions about the direction of $E_{i,t}$'s movement in relation to the sign of the productivity shock. As discussed previously, absent any frictions, a sector receiving a positive productivity shock should witness an increase in capital and labor. However, due to adjustment costs, the actual resource inflow to this sector usually lags behind the optimal level. This likely leads to a decrease in the value of $E_{i,t}$ after

Table 4: Direction of movement in $E_{i,t}$ is correlated with the sign of productivity shocks

Dependent variable $\mathbb{I}(E_{i,t} < E_{i,t-\Delta t})$	$\Delta t = 2$	$\Delta t = 4$	$\Delta t = 6$	$\Delta t = 8$
Dummy indicator of positive accumulative TFP shocks	0.204***	0.214***	0.119***	0.116***
	(0.0251)	(0.0248)	(0.0247)	(0.0250)
N	1593	1593	1593	1593
R^2	0.170	0.196	0.205	0.231
Dummy indicator of positive median TFP shocks	0.195***	0.162***	0.0997***	0.0882***
	(0.0250)	(0.0247)	(0.0240)	(0.0238)
N	1593	1593	1593	1593
R^2	0.167	0.179	0.202	0.227
TFP shocks (accumulative)	0.258***	0.234***	0.133***	0.137***
	(0.0765)	(0.0691)	(0.0492)	(0.0408)
N	1593	1593	1593	1593
R^2	0.110	0.137	0.175	0.210
TFP shocks (median)	0.886***	0.923***	0.591*	0.820**
	(0.305)	(0.288)	(0.316)	(0.367)
N	1593	1593	1593	1593
R^2	0.106	0.133	0.172	0.206

Notes: This table examines the correlation between the sign of productivity shocks and the direction of the movement in $E_{i,t}$. The left-hand-side variable is a dummy variable indicating if there is a decline in $E_{i,t}$ over $[t-\Delta t, t]$. On the right-hand side, we include a dummy variable indicating a positive accumulative or median TFP shock and the actual values of the shocks. All regressions include a set of sector- and year-fixed effects. Robust standard errors are reported and in parentheses.

a positive productivity shock.

We test this prediction in Table 4. The top panel regresses a binary variable $\mathbb{I}(E_{i,t} < E_{i,t-\Delta t})$, indicating a reduction in $E_{i,t}$, on another binary variable, reflecting whether the sector has seen a positive accumulative TFP shock over the time interval $[t-\Delta t, t]$, with sector- and year-fixed effects as controls.[22] A three-year rolling window yields an estimated correlation of 0.2, which slightly diminishes with longer windows but remains highly significant. This positive and significant estimate is consistent with our prediction that a rise in TFP correlates with a reduction in the value of $E_{i,t}$. The second panel substitutes the accumulative TFP shocks with the median value of the TFP shocks over the time interval

[22]The accumulative TFP shock is defined as the sum of TFP shocks $\epsilon_{i,t}$ over the period $[t-\Delta t, t]$.

$[t - \Delta t, t]$ and returns similar outcomes, despite slightly lower point estimates. This pattern is once again validated when using the actual TFP shock values as dependent variables. As shown in the bottom two panels, a greater positive TFP shock (median or accumulative) is associated with an increased likelihood of a decrease in $E_{i,t}$.

In summary, the two exercises in this section provide insights into the movement of each $E_{i,t}$ as the cross-sectional distribution of $E_{i,t}$ determines aggregate allocative efficiency. The magnitude, as well as the direction of changes in $E_{i,t}$, are closely linked to the underlying TFP shocks.

Evidence from factor utilization. Next, we present additional evidence based on factor utilization rates. Recent studies documented that dispersions of capacity utilization rates are an essential contributor to dispersions of the marginal product of capital and labor (Gorodnichenko et al., 2021). Using a model of investment with adjustment costs, Abel and Eberly (1998) showed that factor utilization rates represent the position in the inaction region. As volatility increases, the inaction region expands, resulting in a broader range of utilization rates in the cross-section. Consequently, the two episodes of productivity slowdown should be accompanied by increased dispersion of utilization rates, which we investigate next.

To do so, we first construct measures of factor utilization rates. For capital, a common approach uses energy consumption to calculate utilization-adjusted capital service (Burnside et al., 1995). According to the estimation in Burnside et al. (1995), the actual capital service used, $\hat{K}_{i,t}$, can be written as a Leontief function of capital stock and energy consumption. Using the specification in Oberfield (2013), we let $\hat{K}_{i,t} = \min\{E_{i,t}/b_i, u_{i,t}K_{i,t}\}$, where $u_{i,t}$ is the capital utilization rate, $E_{i,t}$ is energy consumption, and b_i represents the energy intensity of capital usage in each sector determined by its fundamental technology. The difference between growth rates of $\hat{K}_{i,t}$ and $K_{i,t}$, therefore, represents changes in capital utilization rates: $\Delta \log u^k_{i,t} = \Delta \log \hat{K}_{i,t} - \Delta \log K_{i,t}$.

For labor utilization rates, we follow the tradition of growth accounting literature and define the rate as the average hours worked per employed person (Basu et al., 2006; Fernald, 2014). This then allows us to compute changes in labor utilization rate $\Delta \log u^l_{i,t}$ for each sector on a yearly basis. The cross-sectional dispersions in $\Delta \log u^k_{i,t}$ and $\Delta \log u^l_{i,t}$ are used to measure the dispersion in factor utilization rates.[23]

Figure 6: Cross-sectional dispersions of factor utilization rates

(a) Standard deviation **(b)** Inter-quartile range

Notes: This table plots the cross-sectional dispersions in capital and labor utilization rates every year. Panels (a) and (b) plot the standard deviation and inter-quartile range, respectively. The time series are HP-filtered with a smoothing parameter of 6.25.

Figure 6 displays the HP-filtered time-series of the standard deviation (Panel a) and inter-quartile range (Panel b) of the utilization rates every year. Labor utilization dispersion is relatively small and stable, whereas capital utilization dispersion varies significantly over time. Notably, during the 1970s and 2000s, the dispersion of capital utilization rates increases significantly compared to the previous decades. Despite being suggestive, this figure provides further evidence of the mechanism we highlight.

Policy implications. We highlight two key findings in this paper. First, understanding the two major productivity slowdown episodes in the US requires recognizing the importance of the allocative channel. This conclusion holds regardless of the underlying causes of allocative deterioration. Second, we provide evidence for one potential mechanism driving misallocation: increased volatility in the presence of adjustment costs. Although the mech-

[23] A caveat to this analysis is we can only calculate changes in capital utilization rates but not levels; hence we are unable to quantify to what extent utilization rates contribute to the cross-sectional variations, in $E_{i,t}$, as in Gorodnichenko et al. (2021).

anism is inherently dynamic, our analysis shows that examining the time-series properties of allocative efficiency measures and their relationship with the productivity process offers valuable insights into its existence in the data.

These findings emphasize the critical role of volatility in influencing allocation and aggregate productivity growth. According to Asker et al. (2014), worsening allocative efficiency during periods of high volatility arises from firms optimizing production under adjustment costs. In such cases, deviations from the unconstrained optimal allocation, as in our measurement framework, might be constrained-efficient. This means that if the increase in volatility is exogenous and fixed, policies targeting firms' investment and hiring decisions are unlikely to enhance welfare. However, if volatility can be mitigated through better policy, initiatives to reduce it are clearly beneficial for growth and welfare.

The policy implications of our findings therefore hinge on the nature of the volatility increase during slowdown episodes – whether it reflects primitive factors or can be addressed through better policy. While we do not offer a definitive answer, research such as Baker et al. (2016) indicates that heightened economic volatility in the US often coincides with greater policy uncertainty. Based on these findings, it is plausible to interpret the deterioration during slowdown episodes as a rise in misallocation that could be mitigated by fostering a more stable policy environment to reduce economic volatility.

5 Conclusion

This paper quantifies how much of the slowdown in productivity growth can be explained by factor allocation. We apply a tractable decomposition framework to the US economy and show that allocative efficiency explains approximately two-thirds of productivity slowdown in both the 1970s and 2000s. Providing additional evidence on the role of volatility, we find that the slow growth in allocative efficiency in both slowdown episodes is partly due to increased volatility.

Our method may help study other issues related to allocation and growth. One particular direction of work comes to mind. Several countries have experienced fast growth and significant catching up to the frontier in recent decades. The improvement in allocation efficiency has played an essential role in this process (Song et al., 2011; Buera and Shin, 2013). In general, however, there is significant heterogeneity in convergence patterns across countries, even within the group of advanced economies (Cette et al., 2016). On average, developing countries as a whole have not made much progress in closing the income gaps in relation to the US (Johnson and Papageorgiou, 2020). How much of these patterns are explained by variations in allocation efficiency growth across countries? Extending the current framework to a cross-country setting could provide insights into this question.

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Online Appendix

Allocative Efficiency and the Productivity Slowdown

Lin Shao

A Empirics

A.1 Data

Table A.1 shows the sector definition in KLEMS. Sectors highlighted in red are included in our empirical analysis. For the manufacturing sector definition, we include all subsectors listed under the "D Manufacturing" header. The service sector encompasses all subsectors under "G Wholesale and Retail Trade," "I Transportation, Storage and Communication," "K Real Estate, Renting and Business Activities," as well as sectors H, J, M, and N.

Table A.1: List of sectors in KLEMS (2013 version)

- AtB Agriculture hunting forestry and fishing
- C Mining and quarrying
- D Manufacturing
  - 15t16 Food products, beverages and tobacco
  - 17t19 Textiles, textile products leather and footwear
  - 20 Wood and products of wood and cork
  - 21t22 Pulp paper, paper products, printing and publishing
  - 23 Coke refined petroleum products and nuclear fuel
  - 24 Chemicals and chemical products
  - 25 Rubber and plastics products
  - 26 Other non-metallic mineral products
  - 27t28 Basic metals and fabricated metal products
  - 29 Machinery nec
  - 30t33 Electrical and optical equipment
  - 34t35 Transport equipment
  - 36t37 Manufacturing nec; recycling
- E Electricity gas and water supply
- F Construction
- G Wholesale and retail trade
  - 50 Wholesale trade and commission trade except of motor vehicles and motorcycles
  - 51 Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of fuel
  - 52 Retail trade except of motor vehicles and motorcycles; repair of household goods
- H Hotels and restaurants
- I Transport and storage and communication
  - 60t63 Transport and storage
  - 64 Post and telecommunications
- J Financial intermediation
- K Real estate, renting and business activities
  - 70 Real estate activities
  - 71t74 Renting of m&eq and other business activities
- M Education
- N Health and social work

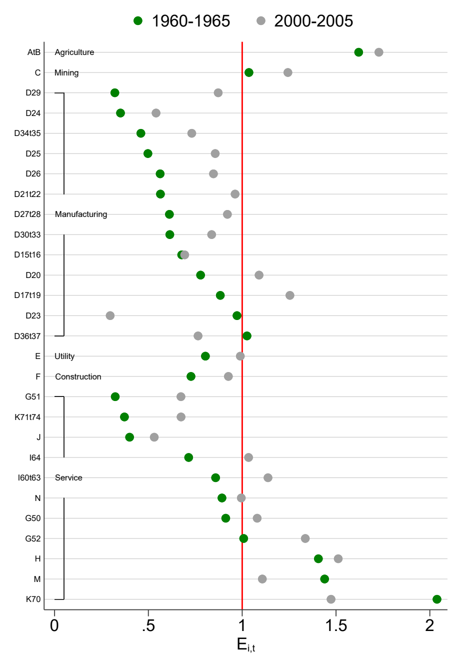
A.2 Improvement in allocative efficiency in the long run

This section delves deeper into the improvement in allocative efficiency over the sample period. Figure A.1 plots the sectoral allocative efficiency $E_{i,t}$, averaged over 1960–65 (beginning of our sample) and 2000–05 (highest overall allocative efficiency years). Recall that E_{it} is the measure of how far away the actual allocation is from the optimal allocation, such that

$$E_{i,t} = \left(\frac{\chi^k_{i,t}}{\chi^{k*}_{i,t}}\right)^{\alpha_{i,t}} \left(\frac{\chi^l_{i,t}}{\chi^{l*}_{i,t}}\right)^{1-\alpha_{i,t}}.$$

If $E_{i,t} > 1$, the actual amount of capital and labor allocated to this sector is above the optimal level. Conversely, if $E_{i,t} < 1$, the actual allocation is below the optimal level. Further, if $\frac{\chi^k_{i,t}}{\chi^{k*}_{i,t}} > 1 (< 1)$, sector i is over-(under-)capitalized. Similarly, $\frac{\chi^l_{i,t}}{\chi^{l*}_{i,t}} > 1 (< 1)$ means that sector i has abundant (not enough) labor.

Figure A.1: Sector-level allocative efficiency $E_{i,t}$, 1960–65 to 2000–05

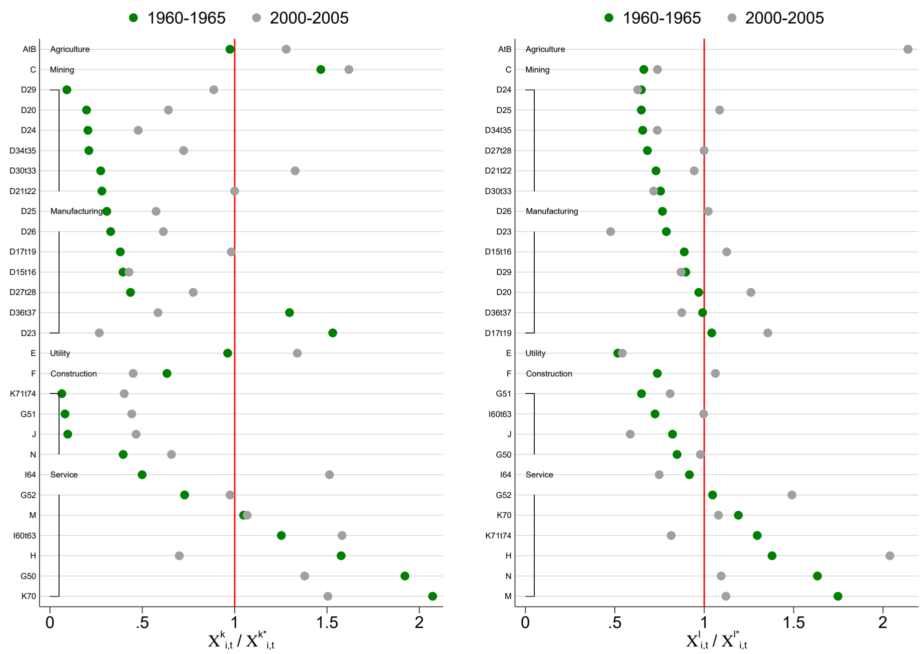


Notes: This figure plots sector-level allocative efficiency measures $E_{i,t}$, averaged over 1960–1965 (green) and 2000–2005 (grey).

Figure A.1 shows that $E_{i,t}$ of manufacturing sectors were almost all below 1 (except for sector D36t37) during 1960–65, indicating that manufacturing sectors needed more capital and labor. In comparison, among service sectors, some had too much capital and labor ($E_{i,t} > 1$) and others did not have enough ($E_{i,t} < 1$). From 1960–65 to 2000–05, $E_{i,t}$ has moved closer to 1 for most sectors in the economy. This means that manufacturing sectors have received more capital and labor ($E_{i,t}$ increased). For the service sectors, this means some sectors received more capital and labor while others gave away capital and labor.

Figure A.2: Sector-level capital and labor allocation, 1960–65 to 2000–05

(a) Capital **(b)** Labor



Notes: This figure plots sector-level measures of the capital ($\frac{\chi^k_{i,t}}{\chi^{k*}_{i,t}}$, Panel a) and labor ($\frac{\chi^k_{i,t}}{\chi^{k*}_{i,t}}$, Panel b) allocative efficiency, averaged over 1960–1965 (green) and 2000–2005 (grey).

Figure A.2 displays the allocation of capital ($\frac{\chi^k_{i,t}}{\chi^{k*}_{i,t}}$) and labor ($\frac{\chi^l_{i,t}}{\chi^{l*}_{i,t}}$) separately for each sector. Not surprisingly, the manufacturing sector's capital was significantly below the optimal level in the early years. Labor was also scarce in manufacturing sectors, but labor allocation was closer to the optimal level than capital.

Taken together, Figures A.1 and A.2 indicate that manufacturing sectors, particularly their capital allocation, were far below optimal in the early years but improved over time, which contributed to a long-term improvement in allocation efficiency.

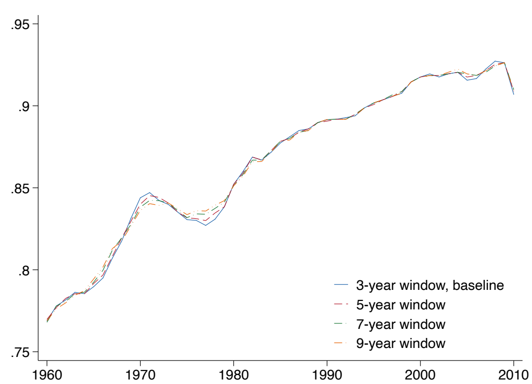
A.3 Identification

A.3.1 Length of the rolling window

Our baseline specification consider a rolling window of 3 years centered in the current period—$[t-1, t, t+1]$ for the period t—and assign the average expenditure shares within this rolling window to the output elasticity in year t. To evaluate the impact of the rolling window length on our analysis, we examine allocative efficiency using rolling windows of 3, 5, 7, and 9 years, as shown in Figure A.3. The results indicate that while a longer rolling window leads to a slightly smaller decline in allocative efficiency, the overall dynamics of measured allocative efficiency remain almost identical across these different specifications.

Table A.2 provides additional support for our findings, showcasing the changes in growth rates of observed labor productivity, fundamental labor productivity, and allocative efficiency compared to previous periods. Importantly, our baseline results remain robust when varying the length of the rolling window. For instance, even with a 9-year rolling window, allocative efficiency explains 7/12 of the observed productivity slowdown in the 1970s and 4/3 of the slowdown in the 2000s.

Figure A.3: Evolution of $\mathbf{E}_t$ over time, alternative rolling windows



Notes: This figure shows the evolution of allocative efficiency. The baseline result is the line with a three-year rolling window.

A.3.2 Biased wedges

Our baseline identification assumption follows Oberfield (2013) and assumes that, on average, the wedges are not biased towards one factor. In this section, we will test the sensitivity of this assumption. More specifically, we ask: what if, on average, the capital wedge is much higher than

Table A.2: Productivity slowdown and the role of allocative efficiency, rolling windows of different lengths

Changes in growth rates compared to the preceding periods (pp)

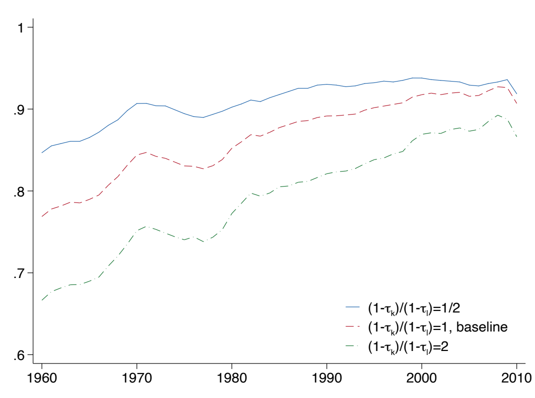
		5-year window		7-year window		9-year window	
Period	data	fundamental	$\mathbf{E}_t$	fundamental	$\mathbf{E}_t$	fundamental	$\mathbf{E}_t$
1960–69	–	–	–	–	–	–	–
1970–79	**-0.12**	**-0.04**	**-0.08**	**-0.04**	**-0.08**	**-0.05**	**-0.07**
1980–89	0.02	-0.03	0.05	-0.02	0.04	-0.02	0.04
1990–99	0.05	0.06	-0.01	0.06	-0.01	0.06	-0.01
2000–07	**-0.03**	**-0.01**	**-0.02**	**-0.01**	**-0.02**	**0.01**	**-0.04**

Notes: This table presents the productivity growth rates under the optimal allocation ($\Delta \log \text{LP}^*$) and the changes in these growth rates from the previous decade ("change") under different lengths of the rolling windows

the labor wedge, or vice versa? We show two cases in which either capital or labor faces larger wedges on average than the other.

In Figure A.4, we show the evolution of $\mathbf{E}_t$ over time under two alternative assumptions. The blue line represents the case in which the capital wedge is higher, $\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 0.5$, for all sectors, the green line represents the other case, in which the labor wedge is higher, $\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 2.0$, and the red line shows the baseline result as a comparison. The figures demonstrate that biased wedges impact the level of allocative efficiency ($\mathbf{E}_t$). A larger capital wedge is associated with a higher $\mathbf{E}_t$, while a larger labor wedge is associated with a lower $\mathbf{E}_t$. However, the trends of $\mathbf{E}_t$ remain similar across all specifications.

Figure A.4: Evolution of $\mathbf{E}_t$ over time, biased wedges



Notes: This figure shows the evolution of allocative efficiency where the wedges are biased on average towards one factor.

The data presented in Table A.3 suggest that when the capital wedge is larger ($\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 0.5$), approximately half of the productivity slowdown witnessed in the 1970s is attributable to allocative efficiency. This ratio decreases to about one-third for the slowdown experienced in the 2000s. In contrast, when the labor wedge is larger ($\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 2.0$), allocative efficiency accounts for a greater proportion of the productivity slowdown, explaining roughly 5/6 and 2/3 of the observed slowdown during the 1970s and 2000s, respectively. The take-away message from this exercise is that, even if the wedges are significantly biased towards one input (capital or labor), allocative efficiency still accounts for at least 1/3 of the observed productivity slowdown.

Table A.3: Productivity slowdown and the role of allocative efficiency, biased wedges

Changes in growth rates compared to the preceding periods (pp)

		$\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 0.5$		$\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 2.0$	
Period	data	fundamental	$\mathbf{E}_t$	fundamental	$\mathbf{E}_t$
1960–69	–	–	–	–	–
1970–79	**-0.12**	**-0.06**	**-0.06**	**-0.02**	**-0.10**
1980–89	0.02	-0.02	0.04	-0.03	0.05
1990–99	0.05	0.07	-0.02	0.06	-0.01
2000–07	**-0.03**	**-0.02**	**-0.01**	**-0.01**	**-0.02**

Notes: This table presents the productivity growth rates under the optimal allocation ($\Delta \log \mathrm{LP}^*$) and the changes in these growth rates from the previous decade ("change") where we assume that wedges are biased on average towards one factor.

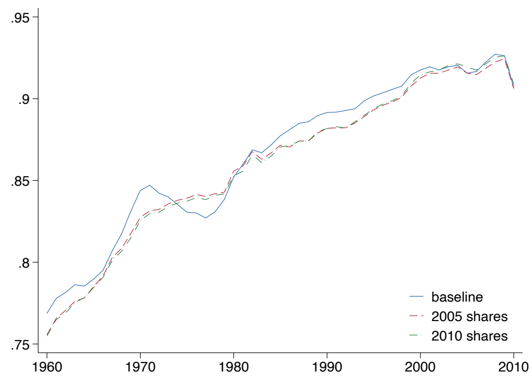
To clarify, our intention is not to suggest that in reality $\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 1/2$ or $\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 2$. The purpose of this exercise is to estimate the impact of biases in wedges on our results. The two quantitative exercises presented assume significant biases in the wedges. For instance, if we assume $\frac{1-\bar{\tau}_k}{1-\bar{\tau}_l} = 1/2$ with $\bar{\tau}_l = 0.2$, it implies $\bar{\tau}_k = 0.6$. In the US data, the capital and labor income shares are approximately 1/3 and 2/3, respectively. Under the assumption of $\bar{\tau}_l = 0.2$ and $\bar{\tau}_k = 0.6$, the true labor and capital income shares would be 4/5 and 1/5, respectively, which significantly deviates from the observed values. By considering these relatively extreme biases, our goal is to provide an upper bound estimation of the potential impact of these assumptions on our results.

A.3.3 Fixed technological parameters

In this section, we consider the identification assumption that technology is hard-wired hence technological parameters are fixed over time. We use data from the later years of our sample to estimate the technology parameters and apply these parameters to the full sample. The choice of using the data from the later years is based on the findings in the literature that in the long run allocative efficiency has improved in the US (Baily et al., 1992; Ziebarth, 2013).

We choose 2010 (the last year in our sample) and 2005 (the last year unaffected by the Great Recession) as the base years for computing the expenditure shares.[24] Figure A.5 demonstrates that the long-run evolution in allocative efficiency and the dynamics during the 2000s are similar under this alternative to the baseline specification. There is also a noticeable deceleration in the improvement of allocative efficiency during the 1970s, although it is less severe than the baseline. Finally, results from Table A.4 show that allocative efficiency can still explain half (6/12) of the observed productivity slowdown during the 1970s and essentially all of the observed slowdown during the 2000s.

Figure A.5: Evolution of $\mathbf{E}_t$ over time, using expenditure shares of later years



Notes: This figure shows the evolution of allocative efficiency where we assume that the factor expenditure shares are undistorted in later years. We pick two base years for this exercise: (i) 2010, the last year of the sample, and (ii) 2005, to avoid the impact of the Great Recession.

A.4 Measurement

A.4.1 Different types of capital and labor

Our empirical analysis relies on the 2013 version of KLEMS. Although the construction of KLEMS data takes into account different compositions of capital types in each sector, one might wonder if this aggregation would cause mismeasurement.[25] The same concern applies to the measurement of labor.

In this section, we repeat the exercise using alternative input measures. We take capital inputs of different types from the 2009 version of KLEMS, which covers a shorter period (1977–2007).

[24] 2005 is the last unaffected year before the Great Recession (which started in 2007) when estimating output elasticities using a three-year rolling window.

[25] See Jorgenson et al. (2014) for details on how the industry-level capital data are constructed.

Table A.4: Productivity slowdown and the role of allocative efficiency, using expenditure shares of later years

Changes in growth rates compared to the preceding periods (pp)

		2005 shares		2010 shares	
Period	data	fundamental	$\mathbf{E}_t$	fundamental	$\mathbf{E}_t$
1960–69	–	–	–	–	–
1970–79	**-0.12**	**-0.06**	**-0.06**	**-0.06**	**-0.06**
1980–89	0.02	0.01	0.01	0.01	0.01
1990–99	0.05	0.04	0.01	0.05	0.00
2000–07	**-0.03**	**0.00**	**-0.03**	**0.00**	**-0.03**

Notes: This table presents the productivity growth rates under the optimal allocation ($\Delta \log \mathrm{LP}^*$) and the changes in these growth rates from the previous decade ("change") where we assume that the factor expenditure shares are undistorted in later years. We pick two base years for this exercise: (i) 2010, the last year of the sample, and (ii) 2005, to avoid the impact of the Great Recession.

Before discussing the results, we first list the asset types provided in the 2009 version of KLEMS. More details can be found in Jorgenson et al. (2014). In this exercise, we consider the most detailed asset classification (eight types) and a broader classification (two types, ICT versus Non-ICT).

- ICT assets
  - Computing equipment
  - Communications equipment
  - Software
- Non-ICT assets
  - Transport equipment
  - Other machinery and equipment
  - Total non-residential investment
  - Residential structures
  - Other assets

In addition, we follow the literature to use labor compensation as a proxy for labor inputs.[26]

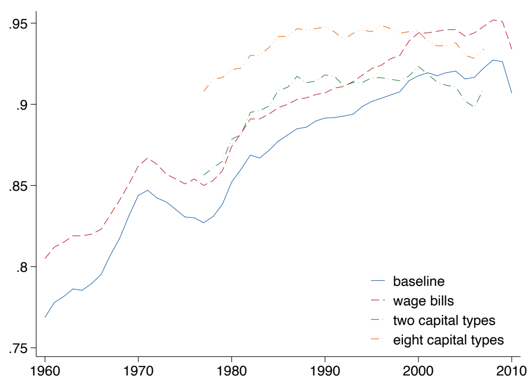
[26] These alternative measures, however, come with their own set of challenges. In constructing the returns to capital by type, KLEMS makes assumptions to split the total capital income into returns of different types, which may introduce measurement errors. Regarding labor inputs, wage bills may address the composition problem, but they cannot distinguish the role of price and quantity in measuring allocative efficiency.

We first extend our theoretical framework to include more than one asset type. More formally, the planner's optimization problem can be written as

$$\max\ Y_t = \prod_{i=1}^{N} Y_{i,t}^{\theta_{i,t}}, s.t.\ Y_{i,t} = A_{i,t} \prod_{s=1}^{S} K_{i,t}^{s\alpha_{i,t}^s} L_{i,t}^{1-\sum_s \alpha_{i,t}^s},\ \forall s,\ \sum_i K_{i,t}^s = K_t^s,\ \sum_i L_{i,t} = L_t,$$

where $s \in \{1, ..., S\}$ represents the different asset types. The optimal allocation of capital and labor is such that $K_{i,t}^{s*} = \chi_{i,t}^{ks*} K_t^s$ and $L_{i,t}^* = \chi_{i,t}^{l*} L_t$, where $\chi_{i,t}^{ks*} = \frac{\theta_{i,t}\alpha_{i,t}^s}{\sum_i \theta_{i,t}\alpha_{i,t}^s}$ and $\chi_{i,t}^{l*} = \frac{\theta_{i,t}(1-\sum_s \alpha_{i,t}^s)}{\sum_i \theta_{i,t}(1-\sum_s \alpha_{i,t}^s)}$. Lastly, the sufficient statistic for allocative efficiency can be written as $\mathbf{E}_t = \prod_{i=1}^{N}\{\prod_s[(\frac{\chi_{i,t}^{ks}}{\chi_{i,t}^{ks*}})^{\alpha_{i,t}^s}](\frac{\chi_{i,t}^{l}}{\chi_{i,t}^{l*}})^{1-\sum_s \alpha_{i,t}^s}\}^{\theta_{i,t}}$.

Figure A.6: Evolution of $\mathbf{E}_t$ over time, alternative measures of inputs



Notes: This figure shows the evolution of allocative efficiency under different measures of capital and labor inputs. The baseline result is the measure with one type of capital and total employment. The other three alternative measures are 1) the use of wage bills (labor compensation) instead to measure labor inputs, 2) the consideration of two types of capital (ICT and non-ICT), and 3) the consideration of eight different types of assets.

Figure A.6 displays the evolution of allocative efficiency under the alternative measures of the capital and labor inputs. The blue line is the benchmark result where we consider only one type of capital and measure labor inputs using employment. Replacing employment with wage bills as measures for labor input, allocative efficiency is slightly higher, but the trend remains very similar to the benchmark result. When considering two types of assets—ICT and non-ICT—the time series only start from the end of the 1970s. The allocative efficiency increases rapidly during the 1980s, stays relatively stable during the 1990s, and starts to decline in the beginning of the 2000s. A similar trend is found when considering eight asset types instead of two, although the level of allocative efficiency is now slightly higher than the two-asset case.

Table A.5 presents the changes in growth rates of observed productivity, fundamental productivity, and allocative efficiency compared to previous decades. The results using wage bills indicate that approximately half of the productivity slowdown in the 1970s and the entire observed slow-

Table A.5: Productivity slowdown and the role of allocative efficiency, alternative measures of inputs

Changes in growth rates compared to the preceding periods (pp)

		two K types		eight K types		wage bills	
Period	data	fundamental	$\mathbf{E}_t$	fundamental	$\mathbf{E}_t$	fundamental	$\mathbf{E}_t$
1960–69	–	–	–	–	–	–	–
1970–79	**-0.12**	–	–	–	–	**-0.06**	**-0.06**
1980–89	0.02	–	–	–	–	-0.02	0.04
1990–99	0.05	0.08	-0.03	0.09	-0.04	0.05	0.00
2000–07	**-0.03**	**-0.01**	**-0.02**	**-0.02**	**-0.01**	**0.00**	**-0.03**

Notes: This table presents the productivity growth rates under the optimal allocation ($\Delta \log \text{LP}^*$) and the changes in these growth rates from the previous decade ("change"). The baseline result is the measure with one type of capital and where labor inputs are measured using employment. The other three alternative measures are 1) the use of wage bills (labor compensation) instead to measure labor inputs, 2) the consideration of two types of capital separately (ICT and non-ICT), and 3) the consideration of eight different types of capital.

down in the 2000s can be attributed to allocative efficiency. Since the capital by type data is only available after 1977, we can only analyze the later slowdown episode. In these instances, allocative efficiency accounts for two-thirds of the productivity slowdown in the two-capital specification and one-third in the eight-capital specification.

To summarize our findings, when extending the analysis to two types of assets, ICT and non-ICT capital, the result is only available from the end of the 1970s. We find a rapid improvement in allocation during the 1980s and a gradual deterioration since the beginning of the 2000s. The exercise with eight types of assets shows a similar trend. In these cases, allocative efficiency accounts for two-thirds of the productivity slowdown in the two-asset specification and one-third in the eight-asset specification. Moreover, by replacing employment with wage bills as a measure of labor input, the level of allocative efficiency is higher, but the trend remains very similar to the baseline result. Our results indicate that approximately half of the productivity slowdown in the 1970s and the entire observed slowdown in the 2000s can be attributed to allocative efficiency.

A.4.2 Non-zero profits

In KLEMS, capital income equals value-added minus labor income; in other words, the underlying assumption is that sectors make zero pure profits. This data treatment is partly motivated by findings that pure profits as a share of value-added output are close to zero in the US (Rotemberg and Woodford, 1995). However, recent studies show that profits shares have been rising in the US, which implies that the data overestimated capital income. Consequently, this leads to an upward bias in capital-output elasticity estimates and capital weights in measuring aggregate allocative efficiency.

Next, we relax the assumption of zero pure profits to test if such an assumption biases our results. We first split the raw capital compensation into pure profits and actual capital income using the estimated profits to capital income ratio of Barkai (2020). The results in Barkai (2020) showed that profits as a share of value-added started to increase at the beginning of the 2000s. Over Barkai's whole sample period (1984–2014), the ratio of profit to capital income reached the highest level of 1/2 in 2007. In 2010, the last year of our sample, the ratio of profit to capital income was approximately 1/5. We repeat our exercises with these two alternative values. More formally, we reestimate the output elasticity in the production functions after taking out profits from capital returns. As a result, this adjustment lowers the output elasticity for capital and increases that for labor.

Figure A.7 shows the changes in measured allocative efficiency under these two alternative specifications. Compared to the baseline results where we assume zero profits, the magnitude of the changes in allocation is generally smaller. However, for the periods of interest, there still exists an apparent stagnation or deterioration in allocation.

Table A.6 presents a more formal evaluation of the role of allocation under the assumption of positive profits. When considering a profit to capital income ratio of 1/5, allocative efficiency accounts for 7/12 of the observed productivity slowdown in the 1970s and 2/3 of the slowdown in the 2000s. With a profit to capital income ratio of 1/2, these ratios slightly decrease to 1/2 and 2/3, respectively. Importantly, our main conclusion remains unchanged: we find that at least half of the productivity slowdown can be attributed to allocative efficiency.

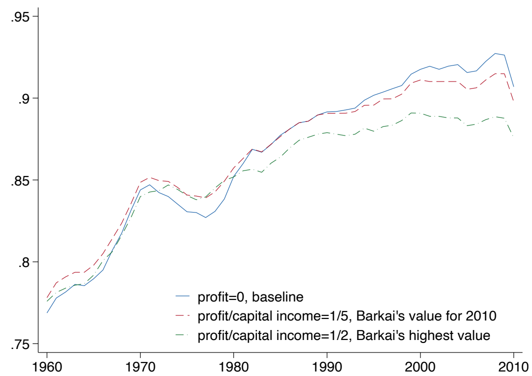
A.5 Extensions of the model: Input-output linkages

This section presents an extension of the baseline model with the incorporation of the input-output linkages. In the input-output economy, each sector $i \in \{1, ..., N\}$ produces good $Q_{i,t}$ using capital, labor, domestic and imported intermediate goods, such that

$$Q_{i,t} = A_{i,t}(K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}} \left(\prod_{j=1}^{N} d_{ij,t}^{\sigma_{ij,t}} \right) \left(\prod_{j=1}^{N} m_{ij,t}^{\lambda_{ij,t}} \right),$$

where $d_{ij,t}$ is the domestic intermediate good j used by sector i, $m_{ij,t}$ is the imported intermediate good j used by sector i, $\sigma_{i,t} = \sum_{j=1}^{N} \sigma_{ij,t}$, and $\lambda_{i,t} = \sum_{j=1}^{N} \lambda_{ij,t}$. There is one final good produced

Figure A.7: Evolution of $\mathbf{E}_t$ over time, non-zero profits



Notes: This figure shows the evolution of allocative efficiency under different profit to capital income ratios. The baseline result is the one with zero profits. The two other values are taken from Barkai (2020): (i) 1/5 for 2010 and (ii) 1/2, the highest point for the period 1984–2014.

Table A.6: Productivity slowdown and the role of allocative efficiency, non-zero profits

Changes in growth rates compared to the preceding periods (pp)

		$\frac{\text{profit}}{\text{capital income}} = \frac{1}{5}$		$\frac{\text{profit}}{\text{capital income}} = \frac{1}{2}$	
Period	data	fundamental	$\mathbf{E}_t$	fundamental	$\mathbf{E}_t$
1960–69	–	–	–	–	–
1970–79	**-0.12**	**-0.05**	**-0.07**	**-0.06**	**-0.06**
1980–89	0.02	-0.02	0.04	0.00	0.02
1990–99	0.05	0.06	-0.01	0.06	-0.01
2000–07	**-0.03**	**-0.01**	**-0.02**	**-0.01**	**-0.02**

Notes: This table presents the productivity growth rates under the optimal allocation ($\Delta \log \text{LP}^*$) and the changes in these growth rates from the previous decade ("change"). In the figure we plot three specifications with different profit to capital income ratios. The baseline result is the one with zero profits. The two other values are taken from Barkai (2020): (i) 1/5 for 2010, and (ii) 1/2, the highest value during his sample period.

by aggregating over these N sectoral goods:

$$Y_t = \prod_i Y_{i,t}^{\theta_{i,t}},$$

where $\sum_{i=1}^{N} \theta_{i,t} = 1$.

The resource constraint on the sectoral good i, therefore, can be written as

$$Q_{i,t} = Y_{i,t} + \sum_{j=1}^{N} d_{ji,t},$$

and the total expenditure on imported goods is

$$X_t = \sum_{i=1}^{N} \sum_{j=1}^{N} \bar{P}_{j,t} m_{ij,t},$$

where $\bar{P}_{j,t}$ is the price of imported intermediate good j relative to the final good.

The planner's problem is to allocate aggregate capital K_t, aggregate labor L_t, sectoral output $Q_{i,t}$ and choose imported intermediate good $m_{ij,t}$ such that the aggregate output net of imports $(Y - X)$ is maximized:

$$\max_{\{K_{i,t}, L_{i,t}, d_{ij,t}, m_{ij,t}\}_{i,j=1}^{N}} \quad Y_t - X_t = \prod_i Y_{i,t}^{\theta_{i,t}} - \sum_{i=1}^{N} \sum_{j=1}^{N} \bar{P}_{j,t} m_{ij,t} \tag{6}$$

$$s.t. \quad Q_{i,t} = A_{i,t} (K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}} \left(\prod_{j=1}^{N} d_{ij,t}^{\sigma_{ij,t}} \right) \left(\prod_{j=1}^{N} m_{ij,t}^{\lambda_{ij,t}} \right),$$

$$Q_{i,t} = Y_{i,t} + \sum_{j=1}^{N} d_{ji,t}, \ \sum_i K_{i,t} = K_t, \ \sum_i L_{i,t} = L_t.$$

The optimal allocation of capital, labor and intermediate goods can be characterized with a set of optimal shares $(\chi_{i,t}^{k*}, \chi_{i,t}^{l*}, \gamma_{ij,t}^{*}, \chi_{i,t}^{y*})$, such that $K_{i,t}^{*} = \chi_{i,t}^{k*} K_t$, $L_{i,t}^{*} = \chi_{i,t}^{l*} L_t$, $d_{ij,t}^{*} = \gamma_{ij,t}^{*} Q_{j,t}^{*}$, $Y_{j,t}^{*} = \chi_{j,t}^{y*} Q_{j,t}^{*}$, and $m_{ij,t}^{*} = (\frac{\theta_{i,t}\lambda_{ij,t}}{\chi_{i,t}^{y*}}) \frac{Y_t^{*}}{\bar{P}_{j,t}}$.[27] The optimal shares can be solved using the following systems of equations:

(i) $\chi_{i,t}^{k*} = \frac{\theta_{i,t}\alpha_{i,t}(1-\sigma_{i,t}-\lambda_{i,t})}{1-\sum_j \gamma_{ji,t}^{*}} / \sum_s \frac{\theta_{s,t}\alpha_{s,t}(1-\sigma_{s,t}-\lambda_{s,t})}{1-\sum_j \gamma_{js,t}^{*}}$, $\forall i \in \{1, ..., N\}$.

(ii) $\chi_{i,t}^{l*} = \frac{\theta_{i,t}(1-\alpha_{i,t})(1-\sigma_{i,t}-\lambda_{i,t})}{1-\sum_{j,t} \gamma_{ji,t}^{*}} / \sum_s \frac{\theta_{s,t}(1-\alpha_{s,t})(1-\sigma_{s,t}-\lambda_{s,t})}{1-\sum_j \gamma_{js,t}^{*}}$, $\forall i \in \{1, ..., N\}$.

[27] See Appendix Section B.1 for details of the solution.

(iii) $\{\chi^{y*}_{i,t}\}_{i=1}^N$ solves the system of equations

$$\frac{1}{\chi^y_{i,t}} = 1 + \frac{1}{\theta_{i,t}} \sum_s \left(\frac{\theta_{s,t}}{\chi^y_{s,t}} \sigma_{si,t} \right), i \in \{1, ..., N\}$$

and

$$\gamma^*_{ij,t} = \frac{\theta_{i,t}\chi^{y*}_{j,t}}{\theta_{j,t}\chi^{y*}_{i,t}} \sigma_{ij,t}.$$

(iv) $\{Q^*_{i,t}\}_{i=1}^N$ solves for the system of equations

$$Q_{i,t} = \chi_{Qi,t} \left(\prod_{s=1}^N Q_{s,t}^{\sigma_{is,t}+\lambda_{i,t}\theta_{s,t}} \right), i \in \{1, ..., N\},$$

where $\chi_{Qi,t} = A_{i,t}[(\chi^{k*}_{i,t}K_t)^{\alpha_{i,t}}(\chi^{l*}_{i,t}L_t)^{1-\alpha_{i,t}}]^{1-\sigma_{i,t}-\lambda_{i,t}}(\prod_{j=1}^N \gamma_{ij,t}^{*\sigma_{ij,t}})[\theta_{i,t}\prod_s(\frac{\chi^{y*}_{s,t}}{\chi^{y*}_{i,t}})^{\theta_{s,t}}]^{\lambda_{i,t}} \prod_{j=1}^N(\frac{\lambda_{ij,t}}{P_{j,t}})^{\lambda_{ij,t}}$.

Allocative efficiency $\mathbf{E}_t$ is the ratio between the output net of imports in the data and that under optimal allocation, that is, $\mathbf{E}_t = \frac{Y_t - X_t}{Y^*_t - X^*_t}$.[28] It can be written as a product of allocative efficiency of capital and labor E^{kl}_t, domestic intermediate goods E^d_t, imported intermediate goods E^m_t, and goods used for the final good production E^y_t:

$$\mathbf{E}_t = E^{kl}_t \cdot E^d_t \cdot E^m_t \cdot E^y_t, \tag{7}$$

(i) $E^{kl}_t = \prod_{i=1}^N(((\frac{\chi^k_{i,t}}{\chi^{k*}_{i,t}})^{\alpha_{i,t}}(\frac{\chi^l_{i,t}}{\chi^{l*}_{i,t}})^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}})^{\sum_n \theta_{n,t}C_{ni,t}}$,

(ii) $E^d_t = \prod_{i=1}^N(\prod_{j=1}^N(\frac{\gamma_{ij,t}}{\gamma^*_{ij,t}})^{\sigma_{ij,t}})^{\sum_n \theta_{n,t}C_{ni,t}}$,

(iii) $E^m_t = \frac{1-\sum_{n=1}^N \frac{\theta_{n,t}\lambda_{n,t}}{\chi^y_{n,t}}}{1-\sum_{n=1}^N \frac{\theta_{n,t}\lambda_{n,t}}{\chi^{y*}_{n,t}}}$,

(iv) $E^y_t = \prod_{n=1}^N(\frac{\chi^y_{n,t}}{\chi^{y*}_{n,t}})^{\theta_{n,t}} \prod_{i=1}^N(\frac{\prod_s(\frac{\chi^y_{s,t}}{\chi^y_{i,t}})^{\theta_{s,t}}}{\prod_s(\frac{\chi^{y*}_{s,t}}{\chi^{y*}_{i,t}})^{\theta_{s,t}}})^{\lambda_{i,t}\sum_n(\theta_{n,t}C_{ni,t})}$,

where C_t is the $N \times N$ Leontief inverse matrix adjusted for imported intermediate goods, such that $C_t = (I - \Omega_t)^{-1}$ and $\Omega_t(i,j) = \sigma_{ij,t} + \lambda_{i,t}\theta_{j,t}$. Details of the model solution can be found in Appendix Section B.2.

[28] Note that the planner optimizes aggregate consumption, C_t, which is defined as $C_t = Y_t - X_t$. In the input-output economy, allocative efficiency is therefore expressed as $\mathbf{E}_t = \frac{C_t}{C^*_t} = \frac{Y_t - X_t}{Y^*_t - X^*_t}$. This expression is different from the value-added economy where $C_t = Y_t$; hence $\mathbf{E}_t = \frac{C_t}{C^*_t} = \frac{Y_t}{Y^*_t}$.

To measure allocative efficiency in the input-output economy, we supplement the KLEMS data with the 2013 version of the World Input-Output Table (WIOT). The KLEMS dataset covers the period of 1947–2010, while the input-output table covers 1995–2011, thus restricting the analysis with input-output linkages to the period of 1995–2010. Below we list all the variables used in the empirical exercise. We distinguish whether each variable is the nominal value ($) or quantity.

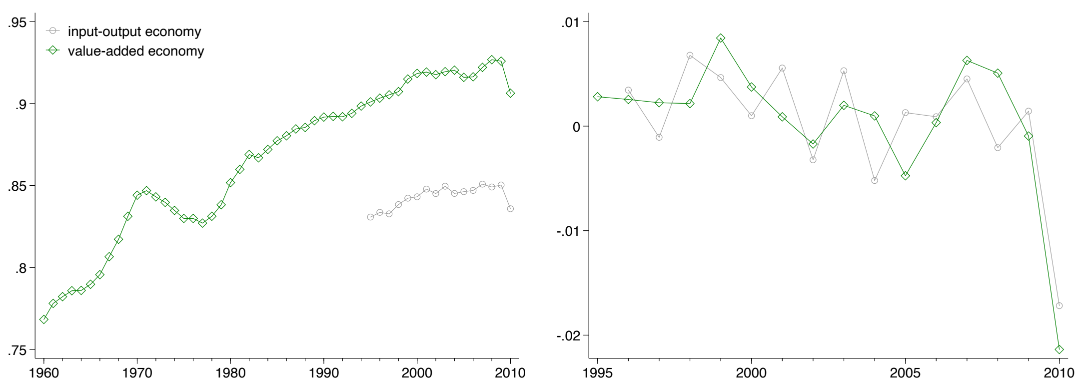
KLEMS (i) sector-level value-added and gross output ($), (ii) sector-level capital and labor compensation, and cost of intermediate goods ($), (iii) sector-level real capital stock, and the number of workers (quantity).

WIOT (i) sector i's use of domestic sector j good ($), (ii) sector i's use of foreign sector j good ($), (iii) sector i good used in the final good production ($).

Figure A.8 panel (a) compares the measured allocative efficiency in the value added economy (baseline) and the input-output economy. Measures of $\mathbf{E_t}$ in the value-added economy are consistently higher than in the input-output economy. However, despite level differences, growth rates of $\mathbf{E_t}$ in these two economies are quite similar to each other during the period when input-output information is available (panel b).

Figure A.8: Evolution of allocative efficiency over time

(a) $\mathbf{E_t}$ **(b)** $\Delta \log(\mathbf{E_t})$



Notes: Panel (a) plots the measured allocative efficiency over time while panel (b) plots the growth rates (as in log difference). The gray lines correspond to the input-output economy and the green lines correspond to the value-added economy.

Table A.7 is analogous to Table 1 in the main text but focuses on the period from 1995 to 2010, when input-output (IO) table information is available. We divide this period into three five-year intervals. In Panel (a), we observe that in the value-added economy (baseline), productivity growth declined by 2 percentage points in the second half of the 2000s. This decline can be fully attributed to slower growth in allocative efficiency. Panel (b) shows identical results for the input-output economy, indicating that incorporating input-output linkages into the model does not alter the results. This aligns with the observation that while input-output linkages influence the measured level of allocative efficiency, they have minimal impact on changes in allocative efficiency over time (Figure A.8).

Table A.7: Slowdown in productivity growth and the role of allocative efficiency

	Growth rates by periods (long log-difference)			Changes in growth rates from preceding period		
	(1)	(2)	(3)	(4)	(5)	(6)
	labor productivity			labor productivity		
Periods	data	"fundamental"	$\mathbf{E}_t$	data	"fundamental"	$\mathbf{E}_t$
(a) VA economy (1995-10)						
1995-99	0.11	0.10	0.01	–	–	–
2000-04	0.11	0.11	0.00	0.00	0.02	-0.0
2005-10	0.10	0.11	-0.01	**-0.02**	**0.00**	**-0.02**
(b) IO economy (1995-10)						
1995-99	0.11	0.10	0.01	–	–	–
2000-04	0.11	0.11	0.00	0.00	0.02	-0.0
2005-10	0.10	0.11	-0.01	**-0.02**	**0.00**	**-0.02**

Notes: Columns (1)-(3) present the growth rates in LP_t (labor productivity, data), LP_t^* (labor productivity, fundamental), and $\mathbf{E}_t$. These growth rates are based on the long differences in labor productivity between the end and the beginning of each period. They are adjusted to reflect the growth rate over periods of the same length (five-year window). Columns (4)-(6) present the changes in these growth rates from the preceding periods. Panel (a) presents results from the value-added economy while panel (b) presents the results from the input-output economy.

A.6 Extensions of the model: CES production system

This section extends the benchmark value-added economy to a more flexible CES production system. The final good is a CES aggregation of intermediate goods: $Y = (\sum_i \omega_i Y_i^{1-\frac{1}{\rho}})^{\frac{\rho}{\rho-1}}$. The intermediate good Y_i is produced using capital and labor, $Y_i = A_i(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}$. In these production functions, $(\omega_i,\ \nu_i)$ are the CES weights and (ρ, ϵ) are the elasticity of substitution. Similar to before, the planner solves the following optimization problem: max $Y, s.t \sum_i K_i = K, \sum_i L_i = L$. Appendix Section B.3 details the model solution.

To evaluate our results in the CES framework, we first estimate the elasticity of substitution parameters, ρ and ϵ, using the following specification:

$$\log \frac{p_{i,t}Y_{i,t}}{p_{N,t}Y_{N,t}} = \beta_\rho \log \frac{Y_{i,t}}{Y_{N,t}} + \chi_i + u_{i,t}, \tag{8}$$

$$\log \frac{R_i K_i}{w_i L_i} = \beta_\epsilon \log \frac{K_i}{L_i} + \chi_i + u_{i,t}, \tag{9}$$

where $\beta_\rho = \frac{\rho}{\rho-1}$, $\beta_\epsilon = \frac{\epsilon}{\epsilon-1}$, χ_i represents the sectoral fixed effects and $u_{i,t}$ is the error term.[29] In Equation (8), $p_{i,t}Y_{i,t}$ is the nominal value-added output for sector $i \in \{1, ..., N\}$ and $Y_{i,t}$ is the corresponding real value-added output. In Equation (9), R_iK_i and w_iL_i are capital and labor income in nominal terms while K_i and L_i are real capital stock and the number of workers in each sector.[30] After obtaining the elasticity of substitution parameters, the CES weights in the production functions can be calculated using the expenditure shares over a rolling window, as in the Cobb-Douglas case.

The point estimate for the elasticity of substitution between sector goods is $\rho = 0.96$, smaller than but not significantly different from one. The elasticity of substitution ρ measures how easy it is for consumers to substitute across a broad set of goods or services. Not surprisingly, the estimates would vary somewhat across different sectoral classification schemes. Aum et al. (2018) categorized all non-agriculture industries into ten broad sectors. Their estimated elasticity was 0.77 across these sectors. Oberfield and Raval (2021) showed that the estimates of elasticity across two-digit manufacturing sectors centered around one from various specifications. In Herrendorf et al. (2013), the benchmark specification estimated the elasticity between broad sectors—agriculture, service, and manufacturing—to be around 0.9. Atalay (2017) suggested that a value smaller but

[29]Both specifications are derived from the cost-minimization conditions following the approach in Aum et al. (2018). The conditions that generate Equations (8) and (9) are $\log(\frac{P_iY_i}{P_NY_N}) = \log(\frac{\omega_i}{\omega_N}) + \frac{\rho-1}{\rho}\log(\frac{Y_i}{Y_N})$ and $\log\frac{R_iK_i}{w_iL_i} = \log\frac{\lambda_i}{1-\lambda_i} + \frac{\epsilon-1}{\epsilon}\log(\frac{K_i}{L_i})$, respectively.

[30]The identification strategy exploits the relationship between changes in input expenditure and changes in input quantity over time. Given the expenditure of inputs, input price and quantity provide essentially the same information. Therefore, the above specification is equivalent to one that regresses expenditure on prices (Atalay, 2017) or regresses quantity on prices (Oberfield and Raval, 2021).

closer to one best characterizes the demand elasticity from consumers. Both Atalay (2017) and Oberfield and Raval (2021) chose elasticity equal to one in their baseline parametrization. Overall, our estimate is within the range of estimates in the literature.

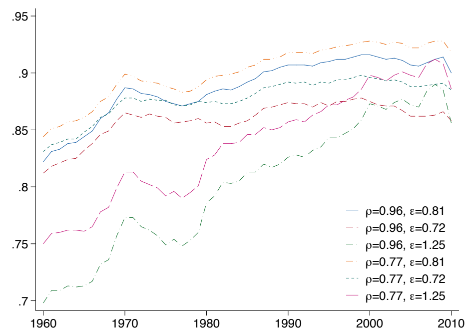
Our estimated elasticity of substitution between capital and labor is $\epsilon = 0.81$, suggesting that capital and labor are gross complements in the sectoral production functions. Several recent papers also estimated the elasticity of substitution between capital and labor at the sectoral/industry level. Among them, Herrendorf et al. (2015) considered three broad sectors, agriculture, manufacturing, and service, and the estimated elasticity of substitution between capital and labor were 1.58, 0.8, and 0.75 in these three sectors, respectively. Alvarez-Cuadrado et al. (2017) found a slightly lower value for the manufacturing (0.78) and service (0.57) sectors. By aggregating up elasticities at the plant level, Oberfield and Raval (2021) obtained an elasticity of 0.72 for manufacturing sectors in 1987 and suggested that it has been trending down since. Aum et al. (2018) combined labor with two types of capital—computer and non-computer—using a nested CES structure and found that the estimated elasticity between computer capital and labor ranged from 1.2 to 1.8.[31]

Given the empirical challenges associated with estimating the elasticity of substitution and the lack of consensus in the literature, applying a range of values for these parameters would be necessary. We set our baseline specification to $\rho = 0.96$ and $\epsilon = 0.81$, which are estimates from our data. We also consider the case of a lower value for the elasticity between sectoral goods, $\rho = 0.77$ (Aum et al., 2018). For the elasticity between capital and labor, we take the estimated value from Oberfield and Raval (2021) (0.72) and from Karabarbounis and Neiman (2014) (1.25), which reflect different views of whether capital and labor are gross substitutes or complements. Therefore we have six combinations of values for ρ and ϵ.

Figure A.9 displays the evolution of $\mathbf{E_t}$ under different values of elasticity for ρ, ϵ. The blue line shows the result under our baseline parametrization where $\rho = 0.96, \epsilon = 0.81$. Two patterns emerge from this figure. First of all, in terms of the level of measured allocative efficiency, this value is in general lower with a higher elasticity of substitution. Among the six combinations of the parameter values, the lowest allocative efficiency occurs with $\rho = 0.96, \epsilon = 1.25$. This pattern is consistent with findings in Epifani and Gancia (2011) and Osotimehin and Popov (2023). Further, the percent changes in allocative efficiency are also larger for the high-elasticity cases. Secondly, for all six cases, there exists a significant deterioration in allocation during the 1970s and either stagnation or deterioration in allocation during the 2000s.

[31] Researchers have also estimated the elasticity of substitution between capital and labor at the aggregate level. There exists a relatively wide range of estimates (see Chirinko, 2008, for a summary). For example, Karabarbounis and Neiman (2014) estimated that capital and labor are gross substitutes with an estimated elasticity between 1.2 and 1.5 using several cross-country aggregate datasets (see also the estimates in Piketty, 2017). On the other hand, most other papers have found that capital and labor are gross complements, including Antràs (2004), Klump et al. (2007), and León-Ledesma et al. (2010), among others.

Figure A.9: Evolution of $\mathbf{E}_t$ over time under different values of elasticity



Notes: This figure shows the evolution of allocative efficiency under different values of elasticity of substitution. The baseline parametrization is $\rho = 0.96, \epsilon = 0.81$, which we estimated directly from the data. We also consider a lower value of $\rho = 0.77$ (Aum et al., 2018). For the value of ϵ, we consider two alternative estimates: 1.25 (Karabarbounis and Neiman, 2014) and 0.72 (Oberfield and Raval, 2021).

Table A.8: Changes in fundamental productivity growth from the previous decade

	CD	CES					
		$\rho = 0.96$			$\rho = 0.77$		
		$\epsilon = 0.81$ baseline	$\epsilon = 0.72$	$\epsilon = 1.25$ highest elas.	$\epsilon = 0.81$	$\epsilon = 0.72$ lowest elas.	$\epsilon = 1.25$
1960–69	–	–	–	–	–	–	–
1970–79	-0.03	-0.04	-0.05	-0.02	-0.05	-0.06	-0.04
1980–89	-0.03	-0.02	-0.01	-0.04	-0.02	0.00	-0.03
1990–99	0.06	0.06	0.06	0.05	0.06	0.06	0.05
2000–07	-0.01	-0.01	-0.01	-0.00	-0.02	-0.01	-0.00

Notes: This table presents the change in fundamental productivity growth compared to the previous decade in the CES framework. The results are presented under six combinations of parameter values for ρ and ϵ, the elasticity of substitution parameters in the production system. The baseline parameterization is $\rho = 0.96$ and $\epsilon = 0.81$. As a comparison, we also include the results from the Cobb-Douglas case, taken from the last column of Table 1.

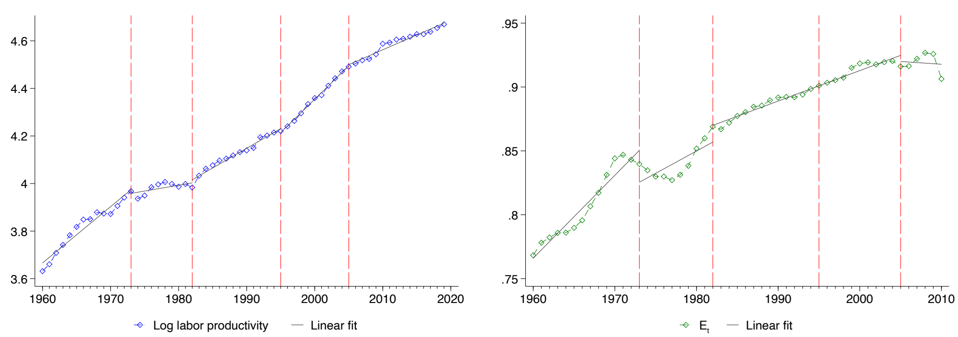
Table A.8 presents the changes in the fundamental productivity growth compared to the previous decade. With the baseline parameterization ($\rho = 0.96, \epsilon = 0.81$), productivity growth in the 1970s and 2000s slows down by 4 pp and 1 pp, respectively. We note that the difference between the baseline CES result and the Cobb-Douglas case is small. Further, we find that higher elasticity is generally associated with a more prominent role for allocative efficiency in explaining the productivity slowdown. For the case with the highest elasticity of substitution ($\rho = 0.96, \epsilon = 1.25$), productivity slowed down by 2 pp during the 1970s. Further, there was no slowdown in productivity during the 2000s. Even for the scenario with the lowest elasticity ($\rho = 0.77, \epsilon = 0.72$), allocation can explain at least half of the productivity slowdown.[32]

A.7 Timing of the slowdown

So far, we have conducted our analysis in decades, focusing on the 1970s and post-2000s as periods of slowdown. However, the timing of the slowdown may not align precisely with these decade-long periods. Indeed, while it is widely recognized in previous papers that the 1970s and 2000s were periods of significant slowdown in productivity, the specific periods studied by these papers vary. For instance, like us, Aum et al. (2018), Decker et al. (2020), and Vandenbroucke (2021) use the decade approach. In comparison, Byrne et al. (2016) argue that the post-2000 slowdown begins in 2005, while the 1970s episode spans 1973–1982.

Figure A.10: Productivity slowdown and allocative efficiency, alternative timing

(a) Labor productivity **(b)** Allocative efficiency



Notes: This figure shows log labor productivity (Panel a) and allocative efficiency measures (Panel b) over time. The time series in both panels are divided by red vertical lines at the years 1973, 1982, 1995, and 2005. The black lines are linear fits of the time series within each of these subperiods.

In this section, we analyze the start and end dates of the slowdown episodes in greater detail.

[32]Our exercise abstracts from the complementarity between intermediate inputs. With the availability of better data, we can extend this framework to incorporate this dimension, thereby potentially enhancing the impact of complementarity (see models in Atalay, 2017, and Osotimehin and Popov, 2023).

To do so, we divide our sample into subperiods based on trends in labor productivity growth. Panel (a) of Figure A.10 displays the log of labor productivity from 1960 to 2019, segmented into five periods: 19601973, 19731982, 19821995, 19952005, and post-2005. These periods align with the slowdown episodes identified in Byrne et al. (2016). Notably, the periods 19731982 and post-2005 are characterized by slower-than-normal labor productivity growth rates, with annual growth rates lower than in preceding periods by 1.88 and 1.08 percentage points, respectively. Panel (b) presents the time series of allocative efficiency $\mathbf{E}_t$, divided into the same subperiods as Panel (a). Similarly, the periods 19731982 and post-2005 experienced slower-than-normal growth in allocative efficiency $\mathbf{E}_t$, with declines of 0.39 and 0.26 percentage points, respectively, compared to the preceding periods.

Table A.9 assesses the role of allocative efficiency in productivity slowdowns. Columns (1)(3) present the average annual growth rates for each subperiod in LP (labor productivity, from data), LP* ("fundamental" productivity), and $\mathbf{E}_t$ (allocative efficiency), calculated as the slopes of the fitted linear functions shown in the previous figure. Columns (4)(6) report the changes in annual growth rates, measured in percentage points relative to the preceding periods. During 19731982 and 20052010, labor productivity growth declined by 1.88 and 1.08 percentage points, respectively, compared to the prior periods. Similarly, allocative efficiency growth also slowed, with declines of 0.39 and 0.26 percentage points during these periods. Consequently, the slower growth in allocative efficiency accounted for 21% (0.39/1.88) and 24% (0.26/1.08) of the decline in labor productivity growth during 19731982 and 20052010, respectively.

Table A.9: Productivity slowdown and the role of allocative efficiency, alternative timing

	Average annual growth rates (percent)			Changes in growth rates from previous period (pp)		
	(1)	(2)	(3)	(4)	(5)	(6)
	labor productivity			labor productivity		
Period	data	"fundamental"	$\mathbf{E}_t$	data	"fundamental"	$\mathbf{E}_t$
1960–73	2.37	1.57	0.80	–	–	–
1973–82	0.49	0.08	0.41	**-1.88**	**-1.49**	**-0.39**
1982–95	1.66	1.37	0.29	1.17	1.29	-0.13
1995–05	2.81	2.60	0.21	1.15	1.22	-0.08
2005–10	1.73	1.77	-0.05	**-1.08**	**-0.82**	**-0.26**

Notes: Columns (1)–(3) display the annual growth rate (percent) in LP_t (data), LP^*_t ("fundamental"), and $\mathbf{E}_t$, calculated by regressing $\log \text{LP}_t$, $\log \text{LP}^*_t$ and $\log \mathbf{E}_t$ on time. Columns (4)–(6) show changes in these growth rates (in percentage points) from the previous periods. The periods highlighted in bold indicate a slowdown in observed labor productivity compared to the previous period.

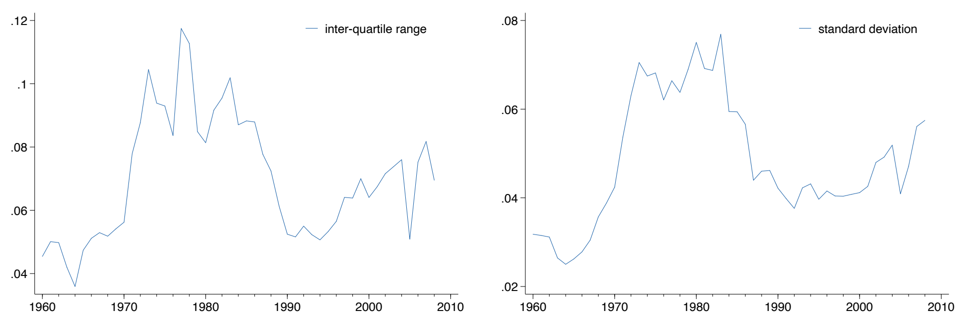
The contribution of allocative efficiency calculated using this alternative timing is smaller than the baseline results for two reasons. First, with this alternative timing, the productivity growth slowdown becomes more prominent. This is especially true for the post-2005 episode, while the subperiod 1995–2005 displays extraordinarily fast productivity growth. In our benchmark analysis, the 2000s decade reflects a blend of these two vastly different periods. Second, the slowdown in allocative efficiency does not coincide perfectly with the slowdown in productivity growth. Particularly in the 1970s episode, this timing difference led to underestimating the role of allocative efficiency.

Lastly, our analysis shows that the timing of the slowdowns in labor productivity and allocative efficiency do not match perfectly. Empirically, the timing of the slowdown in economic activities varies slightly based on which variable we consider. Moreover, the timing of the slowdown in allocative efficiency is sensitive to model specifications. A timing difference can lead to either underestimation or overestimation of allocative efficiencys contribution. For these reasons, we retain the decade approach as our baseline specification for its transparency and include a discussion of the alternative timing as the robustness check.

A.8 Alternative measure of sector-level volatility

In our paper, we follow Bloom et al. (2018) to construct a measure of sector-level volatility ("microeconomic uncertainty"). To do so, we first run a regression for sector-level log TFP: $\log A_{i,t} = \rho \log A_{i,t-1} + \mu_t + \chi_i + \varepsilon_{i,t}$. Then, the variance of $\varepsilon_{i,t}$, the innovation of the productivity process, is a measure of volatility faced by the sectors.

Figure A.11: Dispersion of TFP shock in a rolling 7-year window



Notes: This figure plots the dispersion of the sector-level TFP shock, calculated as the variance of $\varepsilon_{i,t}$, which is the residual term from regression $\log A_{i,t} = \rho \log A_{i,t-1} + \mu_t + \chi_i + \epsilon_{i,t}$. To plot this figure, we first compute, for each sector, the inter-quartile range or the standard deviation of the $\varepsilon_{i,t}$ within each 7-year rolling window centered around the current year. We then compute the median value of the inter-quartile range (Panel a) or the standard deviation (Panel b) across sectors and plot the time series of the median value.

There are two ways to measure the variance of $\varepsilon_{i,t}$. First, it can be calculated as the cross-sector dispersion in ε in any given year. Second, it can be calculated as the dispersion in ε over time for any given sector. In the main text, we use the first approach (see Panel (b) of Figure 5), and, as a robustness check, we apply the second approach in Figure A.11.

More formally, in Figure A.11, we first compute, for each sector, the standard deviation or inter-quartile range of the $\varepsilon_{i,t}$ within each 7-year rolling window centered around the current year. We then compute the median value of the inter-quartile range (Panel a) or the standard deviation (Panel b) across sectors and plot the time series of the median value in Figure A.11. The figures exhibit very similar dynamics as Figure 5, Panel (b) in the main text, particularly regarding the rise in volatility during the 1970s and 2000s.

A.9 Post-2010 dynamics

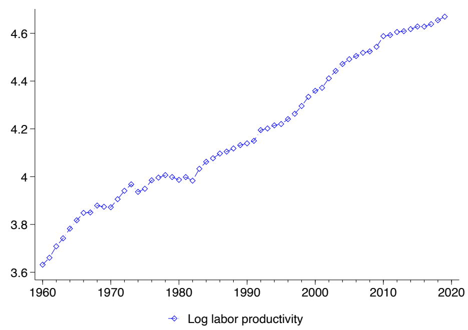
Our baseline analysis uses the 2013 version of the KLEMS database, which only include data up to 2010. However, the slowdown in productivity growth extends beyond 2010. In this section, we examine the post-2010 dynamics using a different version of the KLEMS dataset, published by the Luiss Lab of European Economics (LLEE) in 2021. The LLEE KLEMS dataset has a different industry classification and covers a more recent period up until 2015. However, the LLEE KLEMS has several limitations. For instance, several key variables needed to measure allocative efficiency only became available after 1997. Also, several sectors are missing capital stock information altogether. Keeping in mind these caveats, we use this dataset as a robustness check for the post-2010 dynamics.

Recall that our baseline result finds that $\mathbf{E}_t$, shown as the green line in Panel (b) of Figure A.12, starts to flatten around 2000 after experiencing decades of positive growth. As demonstrated by the red line calculated using the new dataset, this flattening trend continues into the post-2010 period.

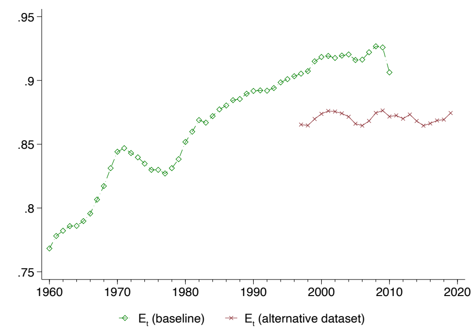
In Table A.10, we extend the baseline results to 2015. The pre-2010 results are based on the KLEMS 2013 data and are identical to Panel (a) of Table 1 in the main text. The post-2010 results are based on the LLEE KLEMS dataset. This table suggests that taking the 2000–15 period as a whole, it exhibits a very similar pattern to the period of 2000–07. Notably, growth rates in allocative efficiency continue to slow, and they can now account for the entirety of the productivity slowdown during this period. Lastly, note that, considering the short length of the time series and the issues of the data sets as discussed, this should be viewed only as suggestive evidence.

Figure A.12: Productivity slowdown and allocative efficiency, post-2010 dynamics

(a) Labor productivity



(b) Allocative efficiency



Notes: This figure shows log labor productivity (Panel a) and allocative efficiency measures (Panel b) over time. The green line in Panel (b) is the same as the baseline result while the red line is based on the KLEMS 2021 data.

Table A.10: Slowdown in productivity growth and the role of allocative efficiency, post-2010 experience

	Growth rates by periods (long log-difference)			Changes in growth rates from preceding period		
	(1)	(2)	(3)	(4)	(5)	(6)
	labor productivity			labor productivity		
Period	data	"fundamental"	$\mathbf{E}_t$	data	"fundamental"	$\mathbf{E}_t$
1960–69	0.24	0.16	0.08	–	–	–
1970–79	0.13	0.13	-0.01	**-0.12**	**-0.03**	**-0.08**
1980–89	0.15	0.10	0.04	0.02	-0.03	0.05
1990–99	0.19	0.16	0.03	0.05	0.06	-0.02
2000–15	0.16	0.17	-0.01	**-0.03**	**0.01**	**-0.03**

Notes: Columns (1)–(3) present the growth rates in LP_t (labor productivity, data), LP_t^* (labor productivity, fundamental), and $\mathbf{E}_t$. These growth rates are based on the long differences in labor productivity between the end and the beginning of each period. They are adjusted to reflect the growth rate over periods of the same length (ten-year window). Columns (4)–(6) present the changes in these growth rates from the preceding periods.

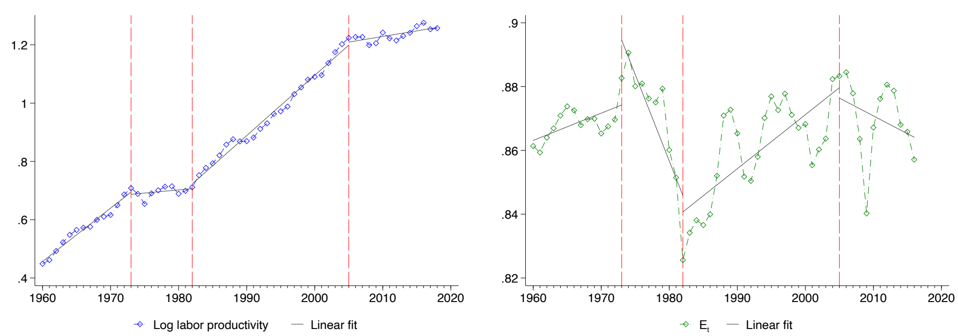
A.10 A more granular look at the manufacturing sector

In this section, we delve deeper into the manufacturing sector, utilizing the NBER-CES Manufacturing Industry Database (1958–2018). This database provides industry-level data for 364 distinct manufacturing industries.[33]

In Figure A.13, Panel (a) shows the evolution of log labor productivity divided into four subperiods: 1960–73, 1973–82, 1982–2005, and post-2005. As there is no significant trend break during 1982–2005, we consider this period of rapid productivity growth as a whole. Moreover, Panel (b) shows the dynamics of $\mathbf{E}_t$, which appears to be noisier than the baseline results, perhaps because the NBER-CES dataset is constructed using firm surveys and has a much finer industry classification. Nevertheless, despite the noise, the figure indicates that 1973–82 and post-2005 have significantly slower growth rates in labor productivity as well as allocative efficiency.

Figure A.13: Manufacturing labor productivity and allocative efficiency over time

(a) Labor productivity **(b)** Allocative efficiency



Notes: This figure shows log labor productivity (Panel a) and allocative efficiency measures (Panel b) over time. The time series in both panels are divided by red vertical lines for the years 1973, 1982, and 2005. The black lines are linear fits of the time series within each of these subperiods.

Table A.11 examines the role of allocative efficiency more formally. Columns (1)–(3) show that the average growth rate in observed labor productivity was only 0.22% and 0.39% during the periods 1973–82 and 2005–16, respectively. At the same time, the annual growth rates in allocative efficiency in these two periods were negative. As shown in Columns (4)–(6), compared with the preceding periods, labor productivity growth rates during 1973–82 and 2005–16 are lower by 1.61 and 1.68 percentage points, respectively, while allocative efficiency growth rates are 0.73 and 0.33 percentage points lower. The lack of improvement in allocative efficiency contributes to 45% (0.73/1.61) and 20% (0.33/1.68) of the slowdown in observed labor productivity growth in these periods.

[33] We use the term manufacturing *industries* to differentiate from the broader manufacturing *sector* that encompasses all these industries.

Table A.11: Manufacturing-sector productivity slowdown: The role of allocative efficiency

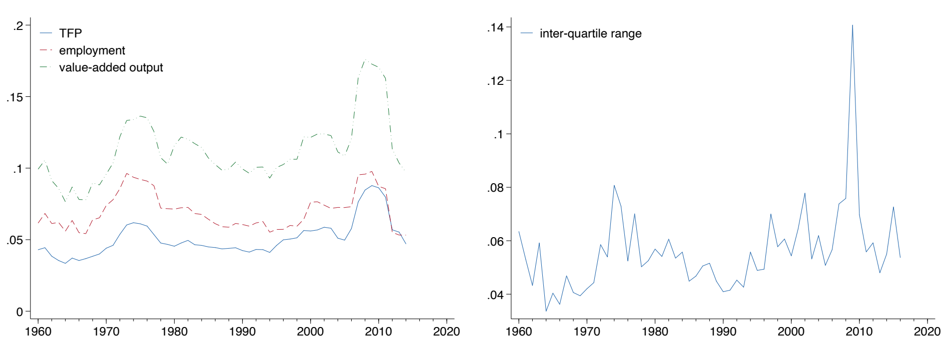
	Average annual growth rates (percent)			Changes in growth rates from previous period (pp)		
	(1)	(2)	(3)	(4)	(5)	(6)
	labor productivity			labor productivity		
Period	data	"fundamental"	$\mathbf{E}_t$	data	"fundamental"	$\mathbf{E}_t$
1960–1973	1.83	1.73	0.10	–	–	–
1973–1982	0.22	0.85	-0.63	**-1.61**	**-0.89**	**-0.73**
1982–2005	2.07	1.87	0.20	1.85	1.02	0.83
2005–2016	0.39	0.51	-0.13	**-1.68**	**-1.36**	**-0.33**

Notes: Columns (1)–(3) display the annual growth rate (percent) in LP_t (data), LP_t^* ("fundamental"), and $\mathbf{E}_t$, calculated by regressing $\log \text{LP}_t$, $\log \text{LP}_t^*$ and $\log \mathbf{E}_t$ on time. Columns (4)–(6) show changes in these growth rates (in percentage points) from the previous periods. The periods highlighted in bold indicate a slowdown in observed labor productivity compared to the previous period.

We construct measures of industry-level volatility using the NBER-CES manufacturing database and follow the methodology in Section 4.3. Panel (a) of Figure A.14 plots the cross-sectional standard deviation of industry-level TFP, employment, and value-added output growth rates. Panel (b) shows the variance of industry-level TFP shocks unforecasted by an AR(1) model. This figure is analogous to Figure 5 in the main text. Similar to Figure 5, we observe elevated volatility during the 1970s and post-2000.

Figure A.14: Volatility of sector-level shocks, manufacturing only

(a) S.D. in sectoral growth rates **(b)** Dispersion in TFP shocks



Notes: These figures are based on the NBER-CES manufacturing database. Panel (a) plots the cross-sectional s.d. of sectoral growth rates in employment, real value-added output and TFP. Panel (b) plots the cross-sectional dispersion (inter-quartile range) in TFP shocks, computed as the residual terms from regression $\log A_{i,t} = \rho \log A_{i,t-1} + \mu_t + \chi_i + \epsilon_{i,t}$.

Table A.12 examines the relationship between the industry-level volatility measures cross-industry allocative efficiency $\mathbf{E}_t$ in the manufacturing sector. More formally, we regress changes in allocative efficiency ($\Delta \log \mathbf{E}_t$) measures on measures of industry-level volatility, measured as the cross-sectional dispersion (IQR) of $\varepsilon_{i,t}$. The estimated relationship between volatility and changes in allocative efficiency is negative and statistically significant at the 5 percent level when controlling for the volatility of $t-1$ and $t-2$. With industry-level volatility, the estimated correlation is around 0.2–0.35. Compared with the estimates of Columns (4)–(6) in Table 2 using the KLEMS data, the estimates found in this table are twice or three times as large.

Table A.12: Relationship between allocative efficiency and volatility measures, manufacturing sector only

	(1)	(2)	(3)
year t	-0.190	-0.347**	-0.351**
	(0.161)	(0.156)	(0.158)
year $t-1$		0.305*	0.287
		(0.179)	(0.195)
year $t-2$			
N	58	57	56
R^2	0.046	0.142	0.142

Notes: This table reports results from regressing log difference in $\mathbf{E_t}$ on the volatility of the industry-level shock of $t, t-1, t-2$. Industry-level volatility is measured as the cross-sectional dispersion (IQR) of $\varepsilon_{i,t}$.

Overall, the dynamics of $\mathbf{E}_t$ appear noisier than the baseline results. Nevertheless, despite the noise, the result indicates that 1973-82 and post-2005 have significantly slower growth rates in labor productivity as well as allocative efficiency. During these periods, 45% and 20% of the slowdown in labor productivity growth can be attributed to a lack of improvement in allocative efficiency. The manufacturing sector experienced elevated levels of volatility during the 1970s and again since 2000. Additionally, both industry and firm-level volatility are associated with declining allocation efficiency. Using this granular dataset and incorporating firm-level volatility, the results confirm and expand upon our baseline findings using the KLEMS dataset.

B Model

B.1 Measuring allocative efficiency in the value-added economy

Solving planner's problem The solution to the planner's problem requires the equalization of MPK and MPL across sectors, such that,

$$\frac{\partial \log Y}{\partial K_i} = \lambda$$

$$\frac{\partial \log Y}{\partial L_i} = \eta.$$

They can be written as

$$K_i^* = \frac{\theta_i \alpha_i}{\lambda}$$

$$L_i^* = \frac{\theta_i (1-\alpha_i)}{\eta}.$$

Given the resource constraint, we get

$$K_i^* = \chi_{i,t}^{k*} K$$

$$L_i^* = \chi_{i,t}^{l*} L,$$

where $\chi_{i,t}^{k*} = \frac{\theta_i \alpha_i}{\sum_i \theta_i \alpha_i}$ and $\chi_{i,t}^{l*} = \frac{\theta_i (1-\alpha_i)}{\sum_i \theta_i (1-\alpha_i)}$.
Q.E.D.

Deriving allocative efficiency The final good output under optimal allocation can be written as

$$\begin{aligned} Y^* &= \prod_i Y_i^{*\theta_i} \\ &= \prod_i (A_i K_i^{*\alpha_i} L_i^{*1-\alpha_i})^{\theta_i} \\ &= \prod_i (A_i (\chi_i^{k*} K)^{\alpha_i} (\chi_i^{l*} L)^{1-\alpha_i})^{\theta_i}. \end{aligned}$$

Similarly, the final output in the data is

$$\begin{aligned} Y &= \prod_i Y_i^{\theta_i} \\ &= \prod_i (A_i K_i^{\alpha_i} L_i^{1-\alpha_i})^{\theta_i} \\ &= \prod_i (A_i (\chi_i^{k} K)^{\alpha_i} (\chi_i^{l} L)^{1-\alpha_i})^{\theta_i}. \end{aligned}$$

As a result,

$$\mathbf{E}_t = \prod_i \left[\left(\frac{\chi_i^{k*}}{\chi_i^k} \right)^{\alpha_i} \left(\frac{\chi_i^{l*}}{\chi_i^l} \right)^{1-\alpha_i} \right]^{\theta_i}.$$

B.2 Measuring allocative efficiency in the input-output economy

Solving planner's problem The planner's problem is

$$C = \prod_{i=1}^{N}(Q_i - \sum_{j=1}^{N} d_{ji})^{\theta_i} - \sum_i \sum_j \bar{P}_j m_{ij}.$$

The FOCs for K_i, L_i, d_{ij}, m_{ij} are

$$\begin{aligned}
\frac{\partial C}{\partial K_i} &= \theta_i \frac{Y^*}{Y_i^*}\frac{Q_i^*}{K_i^*}\alpha_i(1-\sigma_i-\lambda_i) \\
\frac{\partial C}{\partial L_i} &= \theta_i \frac{Y^*}{Y_i^*}\frac{Q_i^*}{K_i^*}(1-\alpha_i)(1-\sigma_i-\lambda_i) \\
\frac{\partial C}{\partial d_{ij}} &= \theta_i \frac{Y^*}{Y_i^*}[\frac{Q_i^*}{d_{ij}^*}\sigma_{ij} - I_{\{i=j\}}] + \theta_j \frac{Y^*}{Y_j^*}[\frac{Q_j^*}{d_{jj}^*}\sigma_{jj} I_{\{i=j\}} - 1] \\
\frac{\partial C}{\partial m_{ij}} &= \theta_i \frac{Y^*}{Y_i^*}\frac{Q_i^*}{m_{ij}^*}\lambda_{ij} - \bar{P}_j.
\end{aligned}$$

The FOC $\frac{\partial C}{\partial d_{ij}} = 0$ implies

$$d_{ij}^* = \frac{\theta_i Y_j^*}{\theta_j Y_i^*}\sigma_{ij} Q_i^*. \tag{10}$$

Therefore,

$$\begin{aligned}
Y_j^* = Q_j^* - \sum_{i=1}^{N} d_{ij}^* &= Q_j^* - \sum_{i=1}^{N}\frac{\theta_i Y_j^*}{\theta_j Y_i^*}\sigma_{ij}Q_i^*, \\
Y_j^*[1 + \frac{1}{\theta_j}\sum_i(\frac{\theta_i Q_i^*}{Y_i^*}\sigma_{ij})] &= Q_j^*.
\end{aligned}$$

Letting $\chi_j^{y*} = \frac{Y_i^*}{Q_i^*}$, $\{\chi_i^{y*}\}_{i=1}^N$ solve the following equations:

$$\frac{1}{\chi_i^{y*}} = 1 + \frac{1}{\theta_i}\sum_s(\frac{\theta_s}{\chi_s^{*y}}\sigma_{si}) \tag{11}$$

or

$$1 - \chi_j^{y*} = \sum_i \sigma_{ij}\frac{\theta_i \chi_j^{y*}}{\theta_j \chi_i^{y*}}.$$

Letting $\gamma_{ij}^* = \frac{\theta_i \chi_j^{y*}}{\theta_j \chi_i^{y*}}\sigma_{ij}$ in Equation (10), then $d_{ij}^* = \gamma_{ij}^* Q_j^*$. The market clearing condition for Q_i^* implies

$$\chi_i^{y*} = 1 - \sum_s \gamma_{si}^*.$$

FOC $\frac{\partial C}{\partial m_{ij}} = 0$ implies

$$m_{ij}^* = \theta_i \frac{Y^*}{Y_i^*} Q_i^* \frac{\lambda_{ij}}{\bar{P}_j} \tag{12}$$

Since

$$Y^* = \prod_i Y_i^{*\theta_i} = \prod_i (\chi_i^* Q_i^*)^{\theta_i},$$

we have

$$m_{ij}^* = \theta_i \prod_s (\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s} \prod_s (Q_s^*)^{\theta_s} \frac{\lambda_{ij}}{\bar{P}_j}. \tag{13}$$

The FOC $\frac{\partial C}{\partial K_i} = 0$ and $\frac{\partial C}{\partial L_i} = 0$ lead to

$$K_i^* = \chi_i^{k*} K \tag{14}$$

$$L_i^* = \chi_i^{l*} L, \tag{15}$$

where

$$\chi_i^{k*} = \frac{\frac{\theta_i \alpha_i (1-\sigma_i-\lambda_i)}{(1-\sum_j \gamma_{ji}^*)}}{\sum_s \frac{\theta_s \alpha_s (1-\sigma_s-\lambda_s)}{(1-\sum_j \gamma_{js}^*)}}, \chi_i^{l*} = \frac{\frac{\theta_i (1-\alpha_i)(1-\sigma_i-\lambda_i)}{(1-\sum_j \gamma_{ji}^*)}}{\sum_s \frac{\theta_s (1-\alpha_s)(1-\sigma_s-\lambda_s)}{(1-\sum_j \gamma_{js}^*)}}. \tag{16}$$

To fully characterize d_{ij} and m_{ij},we need to solve for Q_i. Replacing d_{ij} and m_{ij} in the production function using $d_{ij}^* = \gamma_{ij}^* Q_j^*$ and (13), we get

$$\begin{aligned}
Q_i^* &= A_i (K_i^{*\alpha_i} L_i^{*1-\alpha_i})^{1-\sigma_i-\lambda_i} (\gamma_{i1}^* Q_1^*)^{\sigma_{i1}} \cdots (\gamma_{iN}^* Q_N^*)^{\sigma_{iN}} \prod_{j=1}^N \{\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s} \prod_s (Q_s^*)^{\theta_s} \frac{\lambda_{ij}}{\bar{P}_j}\}^{\lambda_{ij}} \\
&= A_i (K_i^{*\alpha_i} L_i^{*1-\alpha_i})^{1-\sigma_i-\lambda_i} (\prod_{j=1}^N \gamma_{ij}^{\sigma_{ij}}) (\prod_{j=1}^N Q_j^{*\sigma_{ij}}) [\prod_s (Q_s^*)^{\theta_s}]^{\lambda_i} [\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i} \prod_{j=1}^N (\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}} \\
&= A_i [(\chi_i^{k*} K)^{\alpha_i} (\chi_i^{l*} L)^{1-\alpha_i}]^{1-\sigma_i-\lambda_i} (\prod_{j=1}^N \gamma_{ij}^{\sigma_{ij}}) [\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i} \prod_{j=1}^N (\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}} (\prod_{s=1}^N Q_s^{*\sigma_{is}+\lambda_i\theta_s}).
\end{aligned} \tag{17}$$

Define

$$\chi_{Qi}^* = A_i [(\chi_i^{k*} K)^{\alpha_i} (\chi_i^{l*} L)^{1-\alpha_i}]^{1-\sigma_i-\lambda_i} (\prod_{j=1}^N \gamma_{ij}^{\sigma_{ij}}) [\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i} \prod_{j=1}^N (\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}. \tag{18}$$

The above equation can be written as

$$Q_i^* = \chi_{Qi}^* (\prod_{s=1}^N Q_s^{*\sigma_{is}+\lambda_i\theta_s}). \tag{19}$$

Q.E.D.

Deriving allocative efficiency Taking the log of Equation (19) gives $\log Q_i^* = \log\chi_{Qi}^* + \sum_{j=1}^{N}[(\sigma_{ij} + \lambda_i\theta_j)\log(Q_j^*)]$. Let $q^* = [\log(Q_1^*), \ldots, \log(Q_N^*)]'_{N\times 1}$, Equation (19) can be written as

$$q_{N\times 1}^* = b_{N\times 1}^* + \Omega_{N\times N} q_{N\times 1}^*,$$

where $b^*(i) = \log\chi_{Qi}^*$ and $\Omega(i,j) = \sigma_{ij} + \lambda_i\theta_j$.

Therefore, q can be solved as $q = Cb^*$, where $C_{N\times N} = (I - \Omega)^{-1}$ and $Q_n^* = \prod_{i=1}^{N}(\chi_{Qi}^{*C_{ni}})$.

Rewrite Equation (18) as

$$\chi_{Qi}^* = z_i^* K^{\alpha_i(1-\sigma_i-\lambda_i)} L^{(1-\alpha_i)(1-\sigma_i-\lambda_i)},$$

where $z_i^* = A_i[(\chi_i^{k*})^{\alpha_i}(\chi_i^{l*})^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^{N}\gamma_{ij}^{\sigma_{ij}})[\theta_i\prod_s(\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i}\prod_{j=1}^{N}(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}$.

Then Q_n^* can be rewritten as

$$Q_n^* = \prod_{i=1}^{N}(\chi_{Qi}^{*C_{ni}}) = \tilde{A}_n^* K^{\tilde{\alpha}_n} L^{\tilde{\beta}_n}, \tag{20}$$

where $\tilde{A}_n^* = \{\prod_{i=1}^{N} z_i^{*C_{ni}}\}$ and $\tilde{\alpha}_n = \sum_i(\alpha_i(1-\sigma_i-\lambda_i)C_{ni})$, $\tilde{\beta}_n = \sum_i((1-\alpha_i)(1-\sigma_i-\lambda_i)C_{ni})$.

Aggregate output under optimal allocation can be written as a function of aggregate capital K and aggregate labor L:

$$Y^* = \prod_i Y_i^{*\theta_i} = \prod_i(\chi_i^{y*}\tilde{A}_i^* K^{\tilde{\alpha}_i} L^{\tilde{\beta}_i})^{\theta_i} = \bar{A}^* K^{\bar{\alpha}} L^{\bar{\beta}}, \tag{21}$$

where $\bar{A}^* = \prod_{i=1}^{N}(\chi_i\tilde{A}_i^*)^{\theta_i}$ is the aggregate TFP under optimal allocation and $\bar{\alpha} = \sum_n(\tilde{\alpha}_n\theta_n)$, $\bar{\beta} = \sum_n(\tilde{\beta}_n\theta_n)$.

Replacing Q_s^* in Equation (13) using the expression in (20), we can write the expenditure on imported good j as

$$\bar{P}_j m_{ij}^* = [\prod_s(\chi_s^{y*}\tilde{A}_s^*)^{\theta_s}]\{\frac{\theta_i}{\chi_i^{y*}} K^{\sum_s\theta_s\tilde{\alpha}_s} L^{\sum_s\theta_s\tilde{\beta}_s}\}\lambda_{ij} = (\frac{\theta_i\lambda_{ij}}{\chi_i^{y*}})Y^*.$$

The total expenditure on imported goods is

$$X^* = [\sum_{i=1}^{N}(\frac{\theta_i\lambda_i}{\chi_i^{y*}})]Y^*.$$

The output net of imported goods is

$$C^* = Y^* - X^* = Y^*[1 - \sum_{i=1}^{N}(\frac{\theta_i \lambda_i}{\chi_i^{y*}})].$$

Next, we write the data output Y as a function of data allocation (without the stars). The data analog of Equation (17) is

$$Q_i = A_i(K_i^{\alpha_i} L_i^{1-\alpha_i})^{1-\sigma_i-\lambda_i}(\gamma_{i1}Q_1)^{\sigma_{i1}} \cdots (\gamma_{iN}Q_N)^{\sigma_{iN}} \prod_{j=1}^{N}\{\theta_i \prod_s(\frac{\chi_s}{\chi_i})^{\theta_s}\prod_s(Q_s)^{\theta_s}\frac{\lambda_{ij}}{\bar{P}_j}\}^{\lambda_{ij}}. \tag{22}$$

Let $\chi_{Qi} = A_i[(\chi_i^k K)^{\alpha_i}(\chi_i^l L)^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^N \gamma_{ij}^{\sigma_{ij}})[\theta_i \prod_s(\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i}\prod_{j=1}^N(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}$. Equation (22) can be written as

$$Q_i = \chi_{Qi}(\prod_{s=1}^{N} Q_s^{\sigma_{is}+\lambda_i\theta_s}).$$

Let $q = [\log(Q_1), \ldots, \log(Q_N)]'_{N\times1}$, we can solve q as

$$q_{N\times1} = b_{N\times1} + \Omega_{N\times N}q_{N\times1},$$

where $b(i) = \log\chi_{Qi}$ and $\Omega(i,j) = \sigma_{ij} + \lambda_i\theta_j$.

Therefore, q can be solved as $q = Cb$, where $C_{N\times N} = (I-\Omega)^{-1}$.

Then,

$$Q_n = \prod_{i=1}^{N}(\chi_{Qi}^{C_{ni}}) = \tilde{A}_n K^{\tilde{\alpha}_n} L^{\tilde{\beta}_n},$$

where $\tilde{A}_n = \{\prod_{i=1}^N {z_i}^{C_{ni}}\}$ and $\tilde{\alpha}_n = \sum_i(\alpha_i(1-\sigma_i-\lambda_i)C_{ni})$, $\tilde{\beta}_n = \sum_i((1-\alpha_i)(1-\sigma_i-\lambda_i)C_{ni})$.

We can write the data output as

$$Y = \prod_i Y_i^{\theta_i} = \prod_i(\chi_i^y \tilde{A}_i K^{\tilde{\alpha}_i} L^{\tilde{\beta}_i})^{\theta_i} = \bar{A}K^{\bar{\alpha}}L^{\bar{\beta}}, \tag{23}$$

where $\bar{A} = \prod_{i=1}^N(\chi_i^y \tilde{A}_i)^{\theta_i}$ is the aggregate TFP in the data.

In addition, we assume that the expenditure shares of imported intermediate goods are not distorted, such that

$$\bar{P}_j m_{ij} = \lambda_{ij}P_iQ_i = \frac{\lambda_{ij}P_iY_i}{\chi_i^y} = \frac{\theta_i\lambda_{ij}}{\chi_i^y}Y.$$

Thus,

$$X = [\sum_{i=1}^{N} (\frac{\theta_i \lambda_i}{\chi_i^y})] Y$$

and

$$C = Y - X = (1 - \sum_{i=1}^{N} (\frac{\theta_i \lambda_i}{\chi_i^y})) Y.$$

Now we can compute the allocative efficiency as

$$\mathbf{E} = \frac{C}{C^*} = \frac{[1 - \sum_{n=1}^{N} (\frac{\theta_n \lambda_n}{\chi_n^y})] \prod_{n=1}^{N} (\chi_n^y \tilde{A}_n)^{\theta_n} K^{\bar{\alpha}} L^{\bar{\beta}}}{[1 - \sum_{n=1}^{N} (\frac{\theta_n \lambda_n}{\chi_n^{y*}})] \prod_{n=1}^{N} (\chi_n^{y*} \tilde{A}_n^*)^{\theta_n} K^{\bar{\alpha}} L^{\bar{\beta}}},$$

where

$$\frac{\tilde{A}_n}{\tilde{A}_n^*} = \prod_{i=1}^{N} \{ \frac{A_i (\chi_i^{k \alpha_i} \chi_i^{l 1-\alpha_i})^{1-\sigma_i-\lambda_i} (\prod_{j=1}^{N} \gamma_{ij}^{\sigma_{ij}}) [\theta_i \prod_s (\frac{\chi_s^y}{\chi_i^y})^{\theta_s}]^{\lambda_i} \prod_{j=1}^{N} (\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}}{A_i (\chi_i^{k* \alpha_i} \chi_i^{l* 1-\alpha_i})^{1-\sigma_i-\lambda_i} (\prod_{j=1}^{N} \gamma_{ij}^{*\sigma_{ij}}) [\theta_i \prod_s (\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s}]^{\lambda_i} \prod_{j=1}^{N} (\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}} \}^{C_{ni}}$$

$$= \prod_{i=1}^{N} \{ [(\frac{\chi_i^k}{\chi_i^{k*}})^{\alpha_i} (\frac{\chi_i^l}{\chi_i^{l*}})^{1-\alpha_i}]^{1-\sigma_i-\lambda_i} \frac{[\prod_s (\frac{\chi_s^y}{\chi_i^y})^{\theta_s}]^{\lambda_i}}{[\prod_s (\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s}]^{\lambda_i}} \prod_{j=1}^{N} (\frac{\gamma_{ij}}{\gamma_{ij}^*})^{\sigma_{ij}} \}^{C_{ni}}.$$

Rearranged, we get

$$\mathbf{E} = E^{kl} E^d E^m E^y,$$

where

- $E^{kl} = \prod_{i=1}^{N} (((\frac{\chi_i^k}{\chi_i^{k*}})^{\alpha_{i,t}} (\frac{\chi_i^l}{\chi_i^{l*}})^{1-\alpha_i})^{1-\sigma_i-\lambda_i})^{\sum_n \theta_n C_{ni}}$,
- $E^d = \prod_{i=1}^{N} (\prod_{j=1}^{N} (\frac{\gamma_{ij}}{\gamma_{ij}^*})^{\sigma_{ij}})^{\sum_n \theta_n C_{ni}}$,
- $E^m = \frac{1 - \sum_{n=1}^{N} \frac{\theta_n \lambda_n}{\chi_n^y}}{1 - \sum_{n=1}^{N} \frac{\theta_n \lambda_n}{\chi_n^{y*}}}$,
- $E^y = \prod_{n=1}^{N} (\frac{\chi_n^y}{\chi_n^{y*}})^{\theta_{n,t}} \prod_{i=1}^{N} (\frac{\prod_s (\frac{\chi_s^y}{\chi_i^y})^{\theta_s}}{\prod_s (\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s}})^{\lambda_i \sum_n (\theta_n C_{ni})}$.

In addition, we can show that the value-added aggregate production function that features a constant returns to scale. That is, $\bar{\alpha} + \bar{\beta} = 1$. To show this, we only need to show that $(\tilde{\alpha}_n + \tilde{\beta}_n) = 1$. Then it follows that $\bar{\alpha} + \bar{\beta} = \sum_n ((\tilde{\alpha}_n + \tilde{\beta}_n) \theta_n) = \sum_n \theta_n = 1$.

To show that $\tilde{\alpha}_n + \tilde{\beta}_n = 1$, let $B = I - \Omega$. Therefore,

$$\sum_j B(i,j) = \sum_j (1 - (\sigma_i + \lambda_i \theta_j)) = 1 - (\sigma_i + \lambda_i).$$

The first equality is because of the definition of Ω. The second equality holds because $\sum_j \theta_j = 1$. Note that

$$\tilde{\alpha}_n + \tilde{\beta}_n = \sum_i (C_{ni}(1 - \sigma_i - \lambda_i)) = \sum_i \sum_j C(n,i)B(i,j).$$

Since by definition, $BC = CB = I$, $\sum_j \sum_i C(n,i)B(i,j) = 1$ holds for any n. Therefore, $(\tilde{\alpha}_n + \tilde{\beta}_n) = 1$.

B.3 Measuring allocative efficiency in a CES production system

Solving planner's problem Recall the planner's problem:

$$max\ Y = (\sum_i \omega_i Y_i^{1-\frac{1}{\rho}})^{\frac{\rho}{\rho-1}},$$

$$s.t\ \ \sum_i K_i = K, \sum_i L_i = L, Y_i = A_i(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}.$$

FOC wrt K_i is

$$[K_i] : \omega_i Y_i^{*1-\frac{1}{\rho}} = \frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i^{*1-\frac{1}{\varepsilon}}) \eta_K Y^{*-\frac{1}{\rho}},$$

where the multiplier η_k measures the marginal production for capital in each sector.

Summing up the LHS of the above equation over all sectors gives

$$Y^{*\frac{\rho-1}{\rho}} = \sum \omega_i Y_i^{*1-\frac{1}{\rho}} = \sum \frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i^{*1-\frac{1}{\varepsilon}}) \eta_K Y^{*-\frac{1}{\rho}},$$

$$\frac{Y^*}{\sum \frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i^{*1-\frac{1}{\varepsilon}})} = \eta_K = \omega_i Y^{*\frac{1}{\rho}} Y_i^{*-\frac{1}{\rho}} \frac{Y_i^*}{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i^{*1-\frac{1}{\varepsilon}}} \nu_i K_i^{*-\frac{1}{\varepsilon}},$$

$$\omega_i (\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} = \frac{\frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i^{*1-\frac{1}{\varepsilon}})}{\sum_i \frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i^{*1-\frac{1}{\varepsilon}})},$$

where the first equation on the second line follows directly from the first line and the second equation of the second line comes from the FOC w.r.t. K_i. Note that the numerator can be written as

$$\frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}) = \frac{K_i^{*\frac{1}{\varepsilon}-1}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})K_i^*$$

$$= \frac{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}}} K_i^* = \frac{K_i^*}{\frac{\nu_i K_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}}.$$

Let $\alpha_i^* = \frac{\nu_i K_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}$, rewrite the above equation as

$$\frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}) = \frac{K_i^*}{\alpha_i^*}.$$

We next apply a similar approach to labor:

$$\omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} = \frac{\frac{L_i^{*\frac{1}{\varepsilon}}}{1-\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})}{\sum_i \frac{L_i^{*\frac{1}{\varepsilon}}}{1-\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})}.$$

Again, the numerator of the above equation can be written as

$$\frac{L_i^{*\frac{1}{\varepsilon}}}{1-\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}) = \frac{L_i^*}{\frac{(1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}} = \frac{L_i^*}{1-\alpha_i}.$$

To summarize, so far we have derived three equations for the planner's problem,

$$\alpha_i^* = \frac{\nu_i K_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}},\ \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} = \frac{\frac{K_i^*}{\alpha_i}}{\sum_i \frac{K_i^*}{\alpha_i}},\ \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} = \frac{\frac{L_i^*}{1-\alpha_i}}{\sum_i \frac{L_i^*}{1-\alpha_i}}.$$

To further simplify the notation, let $\tilde{K}_i^* = K_i^*/\alpha_i^*$ and $\tilde{L}_i^* = L_i^*/(1-\alpha_i^*)$. Using the last two equations from the line above, we get $\frac{\tilde{K}_i^*}{\tilde{L}_i^*} = \frac{\sum_i \tilde{K}_i^*}{\sum_i \tilde{L}_i^*}$. Further, let $\tilde{K}^* = \sum_i \tilde{K}_i^*$ and $\tilde{L}^* = \sum_i \tilde{L}_i^*$. The last two equations from the line above can be written as

$$K_i^* = \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}}\tilde{K}^*\alpha_i^*;\ L_i^* = \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}}\tilde{L}^*(1-\alpha_i^*).$$

Substitute K_i^* and L_i^* in the sector i's production function:

$$Y_i^* = A_i\{\nu_i[\omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}}\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}}\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}.$$

Therefore,

$$1 = \omega_i(\frac{Y_i^*}{Y^*})^{-\frac{1}{\rho}} \frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*},$$

$$1 = \omega_i^{1-\rho}(\frac{Y_i^*}{Y^*})^{-\frac{1-\rho}{\rho}}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{1-\rho},$$

$$1 = \omega_i^{1-\rho}(\frac{Y_i^*}{Y^*})^{\frac{\rho-1}{\rho}}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{1-\rho},$$

$$\omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1} = \omega_i(\frac{Y_i^*}{Y^*})^{\frac{\rho-1}{\rho}} = \frac{K_i^*}{\tilde{K}^*\alpha_i^*},$$

$$K_i^* = \alpha_i^*\tilde{K}^*\omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1}.$$

Similarly, we apply a similar approach to labor:

$$L_i^* = (1-\alpha_i^*)\tilde{L}^*\omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1}.$$

Let $H_i = A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}$, we can rewrite the above equations:

$$K_i^* = \alpha_i^*\tilde{K}^*\omega_i^{\rho}[\frac{H_i}{Y^*}]^{\rho-1},\ L_i^* = (1-\alpha_i^*)\tilde{L}^*\omega_i^{\rho}[\frac{H_i}{Y^*}]^{\rho-1}.$$

Take them back to sectoral production functions:

$$Y_i^* = A_i(\nu_i K_i^* + (1-\nu_i)L_i^*)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= A_i(\nu_i(\alpha_i^*\tilde{K}^*\omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1})^{1-\frac{1}{\varepsilon}}$$

$$+(1-\nu_i)((1-\alpha_i^*)\tilde{L}^*\omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1})^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1} A_i(\nu_i(\alpha_i^*\tilde{K}^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)((1-\alpha_i^*)\tilde{L}^*\omega_i^{\rho})^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1} A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \omega_i^{\rho} Y^{*1-\rho}\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho}.$$

Take Y_i^* back to the final good production function:

$$Y^* = (\sum_i \omega_i(\omega_i^\rho Y^{1-\rho}\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^\rho)^{1-\frac{1}{\rho}})^{\frac{\rho}{\rho-1}},$$

$$Y^{*\rho-1} = \sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}.$$

Sum K_i^* and L_i^* up across all sectors:

$$K = \sum_i K_i^* = \sum_i \alpha_i^*\tilde{K}^*\omega_i^\rho[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1},$$

$$L = \sum_i L_i^* = \sum_i (1-\alpha_i^*)\tilde{L}^*\omega_i^\rho[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1}.$$

Divide both sides by $\tilde{K}^*$ or $\tilde{L}^*$:

$$\frac{K}{\tilde{K}^*} = \frac{\sum_i \alpha_i^*\omega_i^\rho\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}{\sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}},$$

$$\frac{L}{\tilde{L}^*} = \frac{\sum_i (1-\alpha_i^*)\omega_i^\rho\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}{\sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}.$$

Recall that $\frac{K}{\tilde{K}^*} + \frac{L}{\tilde{L}^*} = 1$. Let $\alpha^* = \frac{K}{\tilde{K}^*}$. Then it follows that $1 - \alpha^* = \frac{L}{\tilde{L}^*}$. Rewrite the above equations with α^*:

$$\alpha^* = \frac{\sum_i \alpha_i^*\omega_i^\rho\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}{\sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}},$$

$$1 = \frac{\sum_i \frac{\alpha_i^*}{\alpha^*}\omega_i^\rho\{A_i\{\nu_i(K\frac{\alpha_i^*}{\alpha^*})^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(L\frac{1-\alpha_i^*}{1-\alpha^*})^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}{\sum_i \omega_i^\rho\{A_i[\nu_i(K\frac{\alpha_i^*}{\alpha^*})^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(L\frac{1-\alpha_i^*}{1-\alpha^*})^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}.$$

Deriving allocative efficiency We have derived the optimal output as

$$Y^* = \{\sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}\}^{\frac{1}{\rho-1}}.$$

Now, replacing $A_i = \frac{Y_i}{(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i)^{\frac{\varepsilon}{\varepsilon-1}}}$ in this equation yields[34]

$$Y^* = \{\sum_i \omega_i^\rho \{ \frac{Y_i}{(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}} [\nu_i (\tilde{K}^* \alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}} \}^{\rho-1} \}^{\frac{1}{\rho-1}}$$

$$= \{\sum_i \omega_i^\rho Y_i^{\rho-1} \{ \frac{[\nu_i (\tilde{K}^* \alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}}{(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}} \}^{\rho-1} \}^{\frac{1}{\rho-1}}.$$

Substitute $Y_i = (\frac{P_i Y_i/\omega_i}{PY})^{\frac{\rho}{\rho-1}} Y$ in the above equation:

$$Y^* = \{\sum_i \omega_i^\rho (\frac{P_i Y_i/\omega_i}{PY})^\rho Y^{\rho-1} \{ \frac{[\nu_i (\tilde{K}^* \alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}}{(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}} \}^{\rho-1} \}^{\frac{1}{\rho-1}}$$

$$\mathbf{E_t} = \frac{Y^*}{Y} = \{\sum_i (\frac{P_i Y_i}{PY})^\rho \{ \frac{\nu_i (\tilde{K}^* \alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}}{\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i^{1-\frac{1}{\varepsilon}}} \}^{\frac{\varepsilon}{\varepsilon-1}(\rho-1)} \}^{\frac{1}{\rho-1}}$$

$$= \{\sum_i (\frac{P_i Y_i}{PY})^\rho \{ \frac{\nu_i (K \frac{\alpha_i^*}{\alpha^*})^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(L \frac{1-\alpha_i^*}{1-\alpha^*})^{1-\frac{1}{\varepsilon}}}{\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i^{1-\frac{1}{\varepsilon}}} \}^{\frac{\varepsilon}{\varepsilon-1}(\rho-1)} \}^{\frac{1}{\rho-1}},$$

where the last equation is derived by replacing $\tilde{K}^*$ with K/α^*.

[34] Note that here Y_i, K_i, L_i are the data, whereas Y_i^*, K_i^*, L_i^* are the optimal allocation derived from the planner's problem.