Allocative Efficiency and the Productivity Slowdown*

Lin Shao† Rongsheng Tang‡

August 26, 2021

Abstract

This paper evaluates the contribution of cross-sector allocative efficiency to the slowdown in productivity growth during the 1970s and the 2000s in the US. We extend the framework of Oberfield (2013) to derive sufficient statistics for allocative efficiency and decompose aggregate productivity growth in a multi-sector economy with or without input-output linkages. We find that approximately two-thirds of the productivity slowdown can be explained by the lack of improvement in allocative efficiency. Data shows that increased volatility is associated with the deterioration of allocation and causes the slowdown in productivity growth.

JEL codes: O47; E23; D57; C67.
Keywords: productivity slowdown; allocative efficiency; volatility; adjustment costs; input-output linkages.

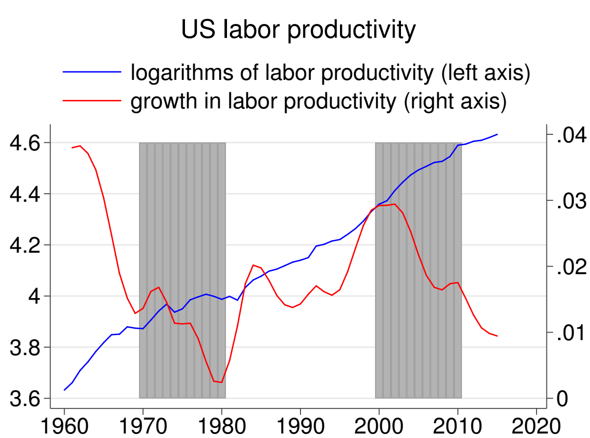
*We thank Susanto Basu, Nicolas Crouzet, Gabriela Galassi, Oleksiy Kryvtsov, Markus Poschke, B. Ravikumar, Diego Restuccia, Ali Sen, Faisal Sohail, Dan Trefler, and audience at CEA 2019, EEA 2020, MEA 2021, World KLEMS Conference, North American Productivity Workshop and the Bank of Canada for helpful comments. The views expressed in this paper are solely those of the authors and may differ from the official Bank of Canada views. Errors are our own.

†Corresponding author, Bank of Canada, lin.j.shao@gmail.com.
‡Shanghai University of Finance and Economics, tang.rongsheng@sufe.edu.cn.

1 Introduction

Growth in real output per worker in the US slowed down significantly in the 1970s and the 2000s (see Figure 1). The slowdown in productivity growth is among the most significant macroeconomic developments in the past few decades and has captured much attention among academic researchers and policymakers. This paper evaluates the role of allocative efficiency—the efficiency with which factors of production are allocated across sectors—in explaining aggregate productivity growth. To do so, we develop a framework to decompose the aggregate productivity growth into changes in allocative efficiency and a residual term which we interpret as changes in fundamental technology. We show that allocative efficiency—or more precisely, the lack of improvement in it—is the common factor behind both episodes of slow productivity growth.

Figure 1: Labor productivity in the US



Notes: This figure plots the logarithms and growth of real output per worker in the United States business sector. The growth rate is computed as the hp-filtered log difference in real output per worker. The growth of real output per hour follows a very similar trend as real output per worker (see Figure 1 of Vandenbroucke, 2019). The gray bars indicate the two productivity slowdown episodes.

Our framework builds on Oberfield (2013) and features (i) a multi-sector value-added economy, and (ii) an input-output economy à la Jones (2013). We characterize the planner's optimal allocation problem and derive sufficient statistics for measuring allocative efficiency. These sufficient statistics capture the deviation of the observed cross-sector allocation of factors of production—capital, labor, and intermediate inputs—from that under the optimal allocation. The empirical exercises rely on the sector-level data in KLEMS and the input-output information collected by World Input-Output Table (WIOT).

We find that allocative efficiency improves gradually over time in the US. On average, allocation efficiency accounts for 20 percent of aggregate productivity growth since the 1960s. However, this contribution varies greatly across decades, with the lowest level occurring in the 1970s and 2000s. To separate the role of allocation from fundamentals, we calculate growth rates under the optimal allocation which essentially eliminates the impact of allocation on growth. We find that productivity slowdown under the optimal allocation is significantly less severe than that observed in the data. By comparing the two growth rates, we find that the lack of improvement in allocation explains approximately two-thirds of the slowdown in productivity growth.

We introduce input-output linkages into the model with the prior that they might affect the level of allocative efficiency as pointed out by previous studies. Indeed, the data shows that our measure of allocative efficiency is lower in the input-output economy than in the value-added economy. However, the changes (growth rates) of the measure are almost identical in these two economies for the period when the input-output information is available. As a result, input-output linkages do not alter our decomposition results significantly because it is the change in efficiency that matters for aggregate growth.

To better understand the dynamics of allocative efficiency, we further decompose its measure to see if it is driven by a particular factor of production or sector. We find that capital seems to be the main driver of change. Additionally, the dynamics of allocative efficiency during those slowdown episodes can be explained by sectors collectively moving away from the optimal allocation. The pattern stands whether we examine the entire distribution of the sectoral allocation measures or aggregate them into broader manufacturing and service sectors.

The measure of allocative efficiency captures the distance between the factor allocation in the data and the optimal allocation. Many factors can cause deviations from optimality, and in this paper, we emphasize the role of volatility. We find that the two productivity slowdown decades are also accompanied by significantly higher volatility at the sector level. In the presence of adjustment costs, higher volatility would lower the responsiveness to individual shocks and cause deterioration in allocation, a mechanism that has been studied in the previous literature (see for example Asker et al., 2014, Bloom et al., 2018 and Decker et al., 2020).

We provide evidence of the correlation between higher volatility and deterioration in allocation by exploiting variations in volatility over time and across sectors. An estimated reduced-form model predicts more than two-thirds of the slowdown in productivity observed in the data. Additionally, in sectors experiencing more volatile TFP shocks, allocative efficiency decreases by a greater amount. Relative to the optimal allocation, the direction of changes in data allocation is also correlated with the TFP shocks in a manner consistent with theoretical predictions. Further, we find evidence for the highlighted mechanism in the increased dispersion of factor utilization rates during the slowdown episodes.

The framework used in this paper relies on parametrical assumptions about the production system. We perform a number of robustness exercises to ensure that our main results are not sensitive to these assumptions. First, we extend the benchmark Cobb-Douglas production system to a CES system. We find that under a range of empirically plausible parameter values, allocation consistently explains a significant portion of the productivity slowdown observed in the data. Another challenge we face is that assumptions are needed to obtain the output elasticities in the production system. These parameters are directly related to factor expenditure shares in the data. However, these expenditure shares may be distorted in the data if, for example, distortions are factor-specific. To deal with this issue, we employ two different specifications to ensure the robustness of the main result. In the first specification, we assume that factor expenditure shares might be distorted in each year but they are undistorted on average over time. The implementation of this strategy makes use of rolling windows of various lengths. The second specification is based on the assumptions that the expenditure shares are undistorted in the later years of the sample, and that their changes are driven by distortions rather than technology. In essence, these two assumptions reflect two different approaches to disentangle technology and distortions in affecting factor intensities: the former permits technical changes across the rolling windows whereas the latter does not.

Finally, we also discuss a few data limitations and examine how they affect the main results. Our data do not report different types of capital and labor at the sector level, nor do they distinguish between capital returns and pure economic profits. To address the first issue, we repeat our analysis using alternative data sources. For capital inputs, we use an early version of the KLEMS dataset that reports capital by type for a shorter period, and we use labor expenditure as a proxy for labor inputs. To address the second issue, we relax the assumption of zero pure profits and attempt to separate pure profits from capital incomes with estimates of profit shares by Barkai (2020). We confirm that the main results are robust to these alternative data treatments.

Related literature There have been many contributions to the study of the US productivity slowdown.[1] Closely related to our paper is Decker et al. (2020). Based on a statistical framework, the paper showed that reallocation across firms slowed down considerably after in 2000s because firms responded less to their idiosyncratic shocks. Similar to us, they also argued that a higher level of volatility—a fact they documentd using firm-level data—has resulted in firms being less responsive to their individual shocks.[2] Our approach differs from

[1]Previous studies of the 1970s slowdown emphasized the oil prices (Jorgenson, 1988), measurement errors (Baily and Gordon, 1988), information technology (Greenwood and Yorukoglu, 1997 and Hornstein and Krusell, 1996), and demography (Feyrer, 2007 and Vandenbroucke, 2019). Measurement errors were also cited as a potential explanation of the post-2000 slowdown, but Byrne et al. (2016) and Syverson (2017) found that they cannot explain a substantial part of the productivity slowdown. Cette et al. (2016) emphasized the important role of a slowdown in the adoption of IT technology. Ramey (2020) argued that the US economy is going through a temporary state of slow technological progress. Bloom et al. (2020) and Gordon (2016), on the other hand, suggested that the slow technological progress might be a permanent state.

[2]Notably, the same mechanism has been studied in other contexts. For example, Bloom (2019) applied it to study fluctuations at the business-cycle frequency.

Decker et al. (2020) by using a direct measure of allocative efficiency to quantify the contribution of allocation to productivity slowdown. In spite of different approaches and data sets, the results in the two papers are in agreement that allocation across firms or sectors played an important role in the productivity slowdown of the 2000s and that higher volatility is a driver behind the undesirable development in allocation efficiency.

Among the papers studying the 1970s productivity slowdown, very few discussed the role of allocation. One exception is Davis and Haltiwanger (2001), who showed that oil price shocks led to higher dispersion in employment growth across manufacturing industries and caused significant job reallocation. Although the focus of the paper was on the reallocation of jobs, the authors also argued that reallocation barriers would lead to a widening of the distance between the actual and desired factor distribution across sectors and output losses. Based on a broader set of sectors, our findings are consistent with this insight. In addition, we provide a measure of the distance between actual and desired factor distribution and document its increase in the 1970s, which we further quantify as a contributing factor to the slowdown in aggregate productivity growth.

Using a multi-sector model similar to ours, some recent studies found that complementarity between inputs caused productivity to slow down in the US as a result of Baumol's diseases (Duernecker et al., 2017 and Aum et al., 2018). Baumol (1967) theorized that in the presence of complementarity, unbalanced productivity growth across sectors can slow down aggregate productivity as resources flow to stagnant sectors. In contrast to the emphasis on allocative efficiency in our paper, Baumol's diseases occur even when resources are allocated optimally across sectors. These papers are complementary to our findings, as they provide an explanation for the slowdown of the fundamental productivity growth, which we do not attempt to explain here.

Our decomposition framework builds on the literature that measure loss from misallocation (Hsieh and Klenow, 2009 and Jones, 2013). In particular, it is a direct extension of the model used in Oberfield (2013).[3] Notably, it relies on parametrical assumptions about the production system, different from the relatively less structural approach taken by Basu and Fernald (2002) and Baqaee and Farhi (2020). On the other hand, our framework has the advantage of providing a global solution rather than relying on linearization around a steady state. Furthermore, these two approaches are based on different notions of allocative efficiency, and thus the results are not entirely comparable to each other (see discussions in Baqaee and Farhi, 2020).

The structure of the paper is as follows: section 2 builds the theoretical framework and section 3 discusses the data as well as the mapping between the model and the data. In section 4, we present the main results. Section 5 includes extensions of the model and robustness checks. Section 6 concludes.

2 Measuring allocative efficiency

This section presents our theoretical framework and discusses how we measure allocative efficiency across sectors in three steps. We first characterize the optimal allocation across sectors as a solution to the planner's problem. We then derive sufficient statistics to measure allocative efficiency using the optimal allocation. Finally, we decompose the aggregate labor productivity growth in the data using the measured allocative efficiency in the previous step.

To begin with, we consider a multi-sector model without input-output linkages in section 2.1, which provides intuition for the methodology and serves as the basis for most of our empirical exercises. We then turn to the economy with input-output linkages in section 2.2. There, it will become clear how the input-output linkages can affect the measurement of allocative efficiency. From here onward, we refer to these two economies as the value-added economy and the input-output economy, respectively.

[3] Similar frameworks have been used to study aggregate productivity growth in countries other than the US, such as Gopinath et al. (2017) and Calligaris et al. (2018).

2.1 Value-added economy

There are N sectors in the economy ($i = \{1, ..., N\}$). In year t, each sector produces a good $Y_{i,t}$ using capital, labor:

$$Y_{i,t} = A_{i,t} K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}},$$

where $A_{i,t}$ is the sectoral productivity. There is one final good Y_t, which is produced by aggregating all sectoral goods, such that

$$Y_t = \prod_{i=1}^{N} Y_{i,t}^{\theta_{i,t}},$$

where $\sum_i \theta_{i,t} = 1$.

Planner's problem The planner's problem is to allocate aggregate capital K_t and labor L_t into the N sectors to maximize the output of final good Y_t:

$$\max \ Y_t = \prod_{i=1}^{N} Y_{i,t}^{\theta_{i,t}}, s.t. \ Y_{i,t} = A_{i,t} K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}}, \ \sum_i K_{i,t} = K_t, \ \sum_i L_{i,t} = L_t. \tag{1}$$

The optimal allocation of capital and labor under problem 1 is such that $K_{i,t}^* = \chi_{i,t}^{k*} K_t$ and $L_{i,t}^* = \chi_{i,t}^{l*} L_t$, where $\chi_{i,t}^{k*} = \frac{\theta_{i,t}\alpha_{i,t}}{\sum_i \theta_{i,t}\alpha_{i,t}}$ and $\chi_{i,t}^{l*} = \frac{\theta_{i,t}(1-\alpha_{i.t})}{\sum_i \theta_{i,t}(1-\alpha_{i,t})}$.

Under the optimal allocation, aggregate capital and labor are allocated to each sector according to optimal shares $\chi_{i,t}^{k*}$ and $\chi_{i,t}^{l*}$. Intuitively, the optimal shares reflect the relative importance of sector i's capital and labor in the production of the final good ($\alpha_i\theta_i$ and $(1-\alpha_i)\theta_i$, respectively).

Allocative efficiency Given the optimal allocation, we can define the allocative efficiency $\mathbf{E}_t$ as the ratio between output in the data (Y_t) and output under the optimal allocation

(Y_t^*), such that $\mathbf{E}_t = \frac{Y_t}{Y_t^*}$, which can be written as the following

$$\mathbf{E}_t = \prod_{i=1}^{N} [(\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}})^{\alpha_{i,t}} (\frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}})^{1-\alpha_{i,t}}]^{\theta_{i,t}}, \tag{2}$$

where $\chi_{i,t}^k = \frac{K_{i,t}}{K_t}$ and $\chi_{i,t}^l = \frac{L_{i,t}}{L_t}$ are the sector i's capital and labor as a share of aggregate K_t and L_t in the data, respectively. Intuitively, $(\frac{\chi_{i,t}^k}{\chi_{i,t}^{k*}})^{\alpha_{i,t}} (\frac{\chi_{i,t}^l}{\chi_{i,t}^{l*}})^{1-\alpha_{i,t}}$ measures sector i's allocative efficiency, which represents the deviation of the observed allocation in the data from the optimal allocation in sector i. Aggregate allocative efficiency $\mathbf{E}_t$ is then simply the weighted geometric mean of sectoral allocative efficiency with sectoral weights θ_i. Details of the model can be found in the appendix section C.

2.2 Input-output economy

In the input-output economy, each sector $i \in \{1, ..., N\}$ produces good $Q_{i,t}$ using capital, labor, domestic and imported intermediate goods, such that

$$Q_{i,t} = A_{i,t}(K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}} (\prod_{j=1}^{N} d_{ij,t}^{\sigma_{ij,t}}) (\prod_{j=1}^{N} m_{ij,t}^{\lambda_{ij,t}}),$$

where $d_{ij,t}$ is the domestic intermediate good j used by sector i, $m_{ij,t}$ is the imported intermediate good j used by sector i, $\sigma_{i,t} = \sum_{j=1}^{N} \sigma_{ij,t}$, and $\lambda_{i,t} = \sum_{j=1}^{N} \lambda_{ij,t}$.[4] There is one final good, produced by aggregating over these N sectoral goods,

$$Y_t = \prod_{i} Y_{i,t}^{\theta_{i,t}},$$

where $\sum_{i=1}^{N} \theta_{i,t} = 1$.

[4] We also studied an input-output economy without imported intermediate inputs and the results are similar to the input-output economy with trade.

The resource constraint on the sectoral good i therefore can be written as

$$Q_{i,t} = Y_{i,t} + \sum_{j=1}^{N} d_{ji,t},$$

and the total expenditure on imported goods is

$$X_t = \sum_{i=1}^{N}\sum_{j=1}^{N} \bar{P}_{j,t} m_{ij,t},$$

where $\bar{P}_{j,t}$ is the price of imported intermediate good j relative to the final good.

Planner's problem The planner's problem is to allocate aggregate capital K_t, aggregate labor L_t, sectoral output $Q_{i,t}$ and choose imported intermediate good $m_{ij,t}$ such that the aggregate output net of imports $(Y - X)$ is maximized,

$$\begin{aligned} \max_{\{K_{i,t},L_{i,t},d_{ij,t},m_{ij,t}\}_{i,j=1}^{N}} \quad & Y_t - X_t = \prod_i Y_{i,t}^{\theta_{i,t}} - \sum_{i=1}^{N}\sum_{j=1}^{N} \bar{P}_{j,t} m_{ij,t} \qquad (3)\\ s.t. \quad & Q_{i,t} = A_{i,t}(K_{i,t}^{\alpha_{i,t}} L_{i,t}^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}}(\prod_{j=1}^{N} d_{ij,t}^{\sigma_{ij,t}})(\prod_{j=1}^{N} m_{ij,t}^{\lambda_{ij,t}}),\\ & Q_{i,t} = Y_{i,t} + \sum_{j=1}^{N} d_{ji,t},\ \sum_i K_{i,t} = K_t,\ \sum_i L_{i,t} = L_t. \end{aligned}$$

The optimal allocation of capital, labor and intermediate goods can be characterized with a set of optimal sectoral shares $(\chi_{i,t}^{k*}, \chi_{i,t}^{l*}, \gamma_{ij,t}^{*}, \chi_{i,t}^{y*})$, such that $K_{i,t}^{*} = \chi_{i,t}^{k*} K_t$, $L_{i,t}^{*} = \chi_{i,t}^{l*} L_t$, $d_{ij,t}^{*} = \gamma_{ij,t}^{*} Q_{j,t}^{*}$, $Y_{j,t}^{*} = \chi_{j,t}^{y*} Q_{j,t}^{*}$, and $m_{ij,t}^{*} = (\frac{\theta_{i,t}\lambda_{ij,t}}{\chi_{i,t}^{y*}})\frac{Y_t^{*}}{\bar{P}_{j,t}}$.[5] The optimal shares can be solved using the following systems of equations:

(i) $\chi_{i,t}^{k*} = \frac{\theta_{i,t}\alpha_{i,t}(1-\sigma_{i,t}-\lambda_{i,t})}{1-\sum_j \gamma_{ji,t}^{*}} / \sum_s \frac{\theta_{s,t}\alpha_{s,t}(1-\sigma_{s,t}-\lambda_{s,t})}{1-\sum_j \gamma_{js,t}^{*}}$, $\forall i \in \{1, ..., N\}$.

(ii) $\chi_{i,t}^{l*} = \frac{\theta_{i,t}(1-\alpha_{i,t})(1-\sigma_{i,t}-\lambda_{i,t})}{1-\sum_{j,t} \gamma_{ji,t}^{*}} / \sum_s \frac{\theta_{s,t}(1-\alpha_{s,t})(1-\sigma_{s,t}-\lambda_{s,t})}{1-\sum_j \gamma_{js,t}^{*}}$, $\forall i \in \{1, ..., N\}$.

[5] See appendix section C for details of the solution.

(iii) $\{\chi_{i,t}^{y*}\}_{i=1}^{N}$ solve the system of equations

$$\frac{1}{\chi_{i,t}^{y}} = 1 + \frac{1}{\theta_{i,t}}\sum_{s}(\frac{\theta_{s,t}}{\chi_{s,t}^{y}}\sigma_{si,t}), i \in \{1, ..., N\},$$

and

$$\gamma_{ij,t}^{*} = \frac{\theta_{i,t}\chi_{j,t}^{y*}}{\theta_{j,t}\chi_{i,t}^{y*}}\sigma_{ij,t}.$$

(iv) $\{Q_{i,t}^{*}\}_{i=1}^{N}$ solve for the system of equations

$$Q_{i,t} = \chi_{Qi,t}(\prod_{s=1}^{N} Q_{s,t}^{\sigma_{is,t}+\lambda_{i,t}\theta_{s,t}}), i \in \{1, ..., N\},$$

where $\chi_{Qi,t} = A_{i,t}[(\chi_{i,t}^{k*}K_t)^{\alpha_{i,t}}(\chi_{i,t}^{l*}L_t)^{1-\alpha_{i,t}}]^{1-\sigma_{i,t}-\lambda_{i,t}}(\prod_{j=1}^{N}\gamma_{ij,t}^{*\sigma_{ij,t}})[\theta_{i,t}\prod_{s}(\frac{\chi_{s,t}^{y*}}{\chi_{i,t}^{y*}})^{\theta_{s,t}}]^{\lambda_{i,t}}\prod_{j=1}^{N}(\frac{\lambda_{ij,t}}{\bar{P}_{j,t}})^{\lambda_{ij,t}}$.

Allocative efficiency Similar as in the value-added economy, $\mathbf{E}_t$ is computed as the ratio between the output net of imports in the data and that under the optimal allocation, that is, $\mathbf{E}_t = \frac{Y_t - X_t}{Y_t^* - X_t^*}$. It can be written as a product of allocative efficiency of capital and labor E_t^{kl}, domestic intermediate goods E_t^{d}, imported intermediate goods E_t^{m}, and goods used for the final good production E_t^{y}:

$$\mathbf{E}_t = E_t^{kl} \cdot E_t^{d} \cdot E_t^{m} \cdot E_t^{y}, \tag{4}$$

(i) $E_t^{kl} = \prod_{i=1}^{N}(((\frac{\chi_{i,t}^{k}}{\chi_{i,t}^{k*}})^{\alpha_{i,t}}(\frac{\chi_{i,t}^{l}}{\chi_{i,t}^{l*}})^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}})^{\sum_n \theta_{n,t}C_{ni,t}}$.

(ii) $E_t^{d} = \prod_{i=1}^{N}(\prod_{j=1}^{N}(\frac{\gamma_{ij,t}}{\gamma_{ij,t}^{*}})^{\sigma_{ij,t}})^{\sum_n \theta_{n,t}C_{ni,t}}$.

(iii) $E_t^{m} = \frac{1-\sum_{n=1}^{N}\frac{\theta_{n,t}\lambda_{n,t}}{\chi_{n,t}^{y}}}{1-\sum_{n=1}^{N}\frac{\theta_{n,t}\lambda_{n,t}}{\chi_{n,t}^{y*}}}$.

(iv) $E_t^{y} = \prod_{n=1}^{N}(\frac{\chi_{n,t}^{y}}{\chi_{n,t}^{y*}})^{\theta_{n,t}}\prod_{i=1}^{N}(\frac{\prod_s(\frac{\chi_{s,t}^{y}}{\chi_{i,t}^{y}})^{\theta_{s,t}}}{\prod_s(\frac{\chi_{s,t}^{y*}}{\chi_{i,t}^{y*}})^{\theta_{s,t}}})^{\lambda_{i,t}\sum_n(\theta_{n,t}C_{ni,t})}$.

where C_t is the $N \times N$ Leontif inverse matrix with adjustment for imported intermediate goods, such that $C_t = (I - \Omega_t)^{-1}$ and $\Omega_t(i,j) = \sigma_{ij,t} + \lambda_{i,t}\theta_{j,t}$. Again, these efficiency measures capture the distance between the optimal allocation ($\chi^{k*}_{i,t}$, $\chi^{l*}_{i,t}$, $\gamma^{*}_{ij,t}$, $\chi^{y*}_{i,t}$) and their data analogs ($\chi^{k}_{i,t}$, $\chi^{l}_{i,t}$, $\gamma_{ij,t}$, $\chi^{y}_{i,t}$). Details of the model can be found in appendix section D.

By simply comparing the sufficient statistics of $\mathbf{E_t}$ in the two economies (equation 2 and 4), we can see that adding input-output linkages alters the measurement of allocation in two important ways: (i) it accounts for the allocation of intermediate inputs, and (ii) the weights representing the importance of each sector now take into account the input-output effects, represented by the Leontief inverse matrix. For the second point, we provide more discussions in appendix section F.

2.3 Decomposition of aggregate productivity in the data

We are now ready to show the decomposition of the aggregate labor productivity in the data using the results from the previous two sections. According to the definition of $\mathbf{E}_t$, the following equation holds: $Y_t = Y^*_t\mathbf{E_t}$. Dividing both sides by the aggregate labor inputs yields $LP_t = LP^*_t\mathbf{E_t}$. Taking the log difference on both side yields $\Delta\log(LP_t) = \Delta\log(LP^*_t) + \Delta\log\mathbf{E_t}$. More formally, aggregate labor productivity in the data, LP_t, can be decomposed into 1) allocative efficiency $\mathbf{E_t}$ and 2) aggregate labor productivity under optimal allocation LP^*_t,

$$LP_t = LP^*_t\mathbf{E_t} \tag{5}$$

$$\Delta\log LP_t = \Delta\log LP^*_t + \Delta\log\mathbf{E_t}. \tag{6}$$

Equation 5 and 6 show the decomposition of the level and the growth rate of labor productivity, respectively. From equation 5, it is clear that allocative efficiency measures the distance of the data (LP) to the production possibility frontier (LP^*).[6] It should also be

[6]See discussions in Baqaee and Farhi (2020) for the different notions of allocative efficiency.

clear from these equations that it is the changes (growth rates) of allocative efficiency, not the levels, that matter for productivity growth. Another useful statistic for our empirical exercise is $\frac{\Delta \log \mathbf{E_t}}{\Delta \log LP_t}$, which measures the contribution of allocative efficiency to the aggregate productivity growth.

3 Application to the US data

Next, we introduce the datasets for our empirical analysis in section 3.1. We then discuss how to map the model to the data in section 3.2.

3.1 Data description

We use the 2013 version of the KLEMS dataset and the World Input-Output Table (WIOT). This version of KLEMS and WIOT are both based on the ISIC Rev. 3 classification thus allow a straightforward mapping of sectors. We restrict our analysis to 28 private sectors in the economy. Table A.1 in the appendix list these sectors. The KLEMS dataset covers the period of 1947–2010, while the input-output table covers 1995–2011, thus restricting the analysis with input-output linkages to the period of 1995–2010. Below we list all the variables used in the empirical exercise. For each variable, we distinguish whether it is the nominal value ($) or quantity.

KLEMS (i) sector-level value-added and gross output ($), (ii) sector-level capital and labor compensation, and cost of intermediate goods ($), (iii) sector-level real capital stock, and the number of workers (quantity).[7]

WIOT (i) sector i's use of domestic sector j good ($), (ii) sector i's use of foreign sector j good ($), (iii) sector i good used in the final good production ($).

[7]The 2013 version of KLMES reports a capital quantity index for each year. An early vintage of EU-KLEMS (2009 version) contains real capital stock based on 1995 price but the data is only available for a shorter time series. We construct real capital stock for all years using the 1995 real capital stock and the quantity index.

Before moving on, some discussions about the datasets are in order. On the one hand, the nature of the datasets limits our attention to cross-sector allocation; hence our analysis leaves out the within-sector allocation dimension. On the other hand, these datasets also feature several important advantages. Compared to firm-level data, KLEMS and WOIT generally cover a longer period and a broader set of sectors. This allows us to study questions that can not be answered due to the lack of high-quality firm-level data—one such example is the 1970s productivity slowdown in the US. In addition, the variables in KLEMS, such as capital stock and output, are constructed from national accounts data, which arguably suffer less from measurement errors. Overall, making use of these datasets allows us to study these two major productivity slowdown episodes in a unified framework without too much concern for the data quality issues.

3.2 Mapping between model and data

In this section, we explain the mapping between the model and the data. Recall that to calculate $\mathbf{E}_t$, we need to compute (i) the allocation of capital, labor and intermediate inputs across sectors in the data $(\chi^k_{i,t}, \chi^l_{i,t}, \chi^y_{i,t}, \gamma_{ij,t})$, and (ii) the optimal allocation $(\chi^{k*}_{i,t}, \chi^{l*}_{i,t}, \chi^{y*}_{i,t}, \gamma^*_{ij,t})$, which in turn requires output elasticities in the production functions $(\alpha_{i,t},\ \sigma_{ij,t},\ \lambda_{ij,t},\ \theta_{i,t})$.

Cross-sector allocation in the data First, we calculate the data allocation of capital, labor and intermediate inputs—$\chi^k_{i,t}, \chi^l_{i,t}, \chi^y_{i,t}, \gamma_{ij,t}$. Ideally, we would like to use the quantity of the inputs to calculate the allocation across sectors. We are able to do so for capital and labor, such that $\chi^k_{i,t} = \frac{K_{i,t}}{\sum_i K_{i,t}}$ and $\chi^l_{i,t} = \frac{L_{i,t}}{\sum_i L_{i,t}}$ where $K_{i,t}$ is the real capital stock and $L_{i,t}$ is the number of workers in sector i. Due to the lack of quantity measure of intermediate inputs, $\gamma_{ij,t}$ and $\chi^y_{i,t}$ are computed using expenditure, such that $\gamma_{ij,t} = \frac{\$d_{ij,t}}{\$Q_{j,t}}$ and $\chi^y_{j,t} = \frac{\$Y_{j,t}}{\$Q_{j,t}}$ where $\$d_{ij,t}$ is sector i's use of sector j good, $\$Q_{j,t}$ is sector j's gross output and $\$Y_{j,t}$ is sector j good used in final good production, all nominal values.

Output elasticities in the production functions Next, we need the output elasticities in the production functions ($\alpha_{i,t}$, $\sigma_{ij,t}$, $\lambda_{ij,t}$, $\theta_{i,t}$) to compute the optimal allocation. Among these parameters, $\theta_{i,t}$ are the stand-in for households' preference. Hence, they are equal to the expenditure share of sector i good in total households' consumption and investment in the data. Similar to Bils et al. (2020), we allow θ_t to vary by year to capture changes in the demand system over time. The rest of the parameters are closely linked to the factor intensities in each sector. Without distortions, or if the distortions are not factor-specific, the factor expenditure shares would reflect the underlying production technology. However, these expenditure shares might be distorted in the data. To deal with this issue, we conduct our analysis with two specifications.

We first follow the strategy in Oberfield (2013) and assume that the factor shares might be distorted in each year but they are undistorted on average over time. Note that this assumption does not mean that on average there are no distortions at the sector level, just that the distortions are not factor-specific. To this end, in each year, we compute the nominal factor expenditure shares and compute the average of these shares in a rolling window centered around the current year. Our results are not sensitive to the length of the rolling window and we report the results from the rolling window of three years in the main text and include more results in the robustness section 5.2.

While the above assumption allows changes in technology across different rolling windows over time, in the second specification, we assume instead that the changes in factor intensities in the data are driven by distortions only. Moreover, we assume that the later years' factor expenditure shares are undistorted while the early years might be distorted. Therefore the cost share of the later years reflect the fundamental technology and therefore can be used to back out the parameters in the production functions. Our main results are found to be robust to this alternative specification and more details can be found in section 5.2.

4 Results

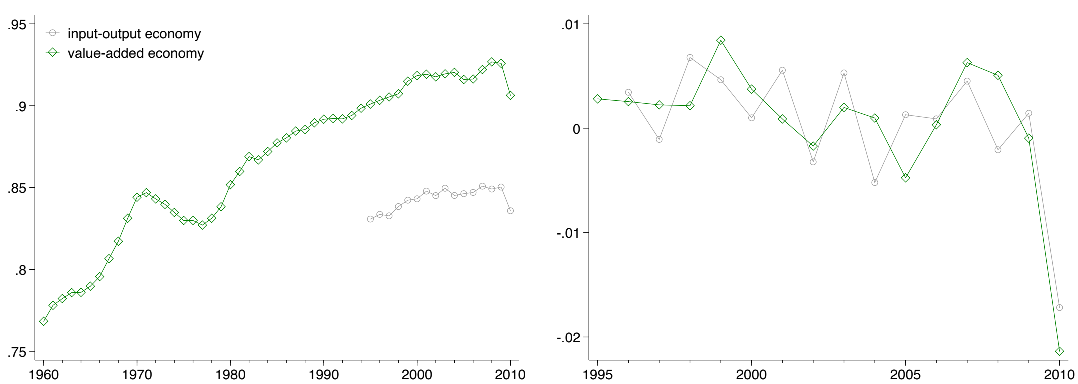
This section presents the main results of the paper. We begin by examining the role of allocative efficiency during the two productivity slowdown episodes in section 4.1. After establishing the quantitative importance of allocation, we proceed to examine the drivers behind the evolution of allocative efficiency (section 4.2).

4.1 Contribution of allocation to productivity slowdown

To quantify the role played by allocative efficiency, we first study how it evolves over time. As shown in figure 2 (a), there is a gradual improvement in allocation since the beginning of the 1960s, indicated by $\mathbf{E_t}$ moving consistently towards 1. There are two exceptions to this trend. At the beginning of the 1970s, allocative efficiency begins to decline, and the deterioration continues throughout the decade; and then after two decades of improvement, it plateaus during the 2000s. Intuitively, even if all else were equal, aggregate productivity growth during these two decades would have been muted because of the undesirable development in allocation efficiency.

Figure 2: Evolution of allocative efficiency over time

(a) $\mathbf{E_t}$ **(b)** $\Delta \log(\mathbf{E_t})$



Notes: Panel (a) plots the measured allocative efficiency over time while panel (b) plots the growth rates (as in log difference). The gray lines correspond to the input-output economy and the green lines correspond to the value-added economy.

The two lines in figure 2 correspond to the two economies and a comparison between the two lines reveals the role played by the input-output linkages. As shown in panel (a), measures of $\mathbf{E_t}$ in the value-added economy are consistently higher than those in the input-output economy. Despite the level differences, the growth rates of $\mathbf{E_t}$ in these two economies are quite similar to each other during the period when the input-output information is available (1995-2010, see panel b).

Table 1: Contribution of allocative efficiency to productivity growth

	$\Delta \log LP_t$	$\Delta \log \mathbf{E_t}$	$\frac{\Delta \log \mathbf{E_t}}{\Delta \log LP_t}$
(a) value-added economy			
1960–69	0.24	0.08	0.32
1970–79	0.13	-0.01	-0.05
1980–89	0.15	0.04	0.30
1990–99	0.19	0.03	0.13
2000–07	0.16	0.01	0.03
1960–2007	0.89	0.18	0.20
(b) input-output economy (95-10)			
1995–99	0.11	0.01	0.13
2000–04	0.11	0.00	0.02
2005–10	0.10	-0.01	-0.13
1995-2010	0.37	0.01	0.02
(c) value-added economy (95-10)			
1995–99	0.11	0.01	0.13
2000–04	0.11	0.00	0.03
2005–10	0.10	-0.01	-0.10
1995–2010	0.37	0.01	0.02

Notes: This table shows the growth rates (log difference) in labor productivity ($\Delta \log LP_t$) and allocative efficiency ($\Delta \log \mathbf{E_t}$) as well as the contribution of allocation to productivity growth $\frac{\Delta \log LP_t}{\Delta \log \mathbf{E_t}}$. Panel (a) and (c) are results from the value-added economy while panel (b) comes from the results of the input-output economy.

More formally, how much does allocation contribute to the aggregate productivity growth? We first look into the value-added economy in table 1 (a), where we present the growth rates in productivity, allocative efficiency, and the contribution of allocation to growth by decade. The results suggest that improvement in allocation accounts for approximately 20 percent of aggregate productivity growth over the period 1960–2007. However, allocation's contribution varies greatly across decades, with the highest levels occurring in the 1960s (32 percent) and 1990s (30 percent) and the lowest levels occurring in the 1970s (-5 percent) and 2000s (3 percent). In panel (b), we move on to analyzing the input-output economy from 1995 to 2010 and divide it up into three five-year subperiods. Allocation is responsible for 13 percent of the productivity growth during the second half of the 1990s, but its contribution drops considerably during the 2000s, reaching -13 percent at the end of the sample. The pattern is confirmed in panel (c) with the results from the value-added economy. In general, we find that allocation contributes little to aggregate productivity growth in the 1970s and 2000s or even has a negative effect.

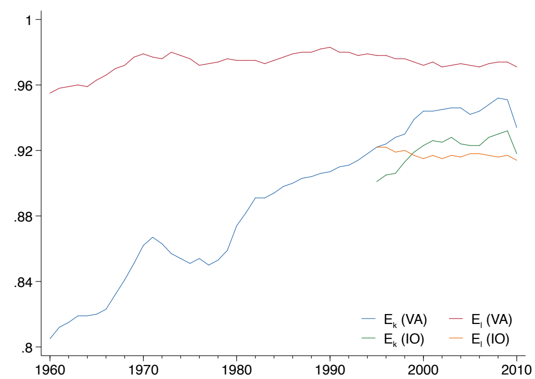
Table 2: Productivity growth in the data and under optimal allocation

	data		optimal	
	dy/y	$\Delta dy/y$	dy/y	$\Delta dy/y$
(a) value-added economy				
1960–69	0.24		0.16	
1970–79	0.13	-0.12	0.13	-0.03
1980–89	0.15	0.02	0.10	-0.03
1990–99	0.19	0.05	0.16	0.06
2000–07	0.16	-0.03	0.16	-0.01
(b) input-output economy (95-10)				
1995–99	0.11		0.10	
2000–04	0.11	0.00	0.11	0.02
2005–10	0.10	-0.02	-0.11	0.00
(c) value-added economy (95-10)				
1995–99	0.11		0.10	
2000–04	0.11	0.00	0.11	0.02
2005–10	0.10	-0.02	-0.11	0.00

Notes: This table shows the growth rate and the changes in the growth rate of labor productivity for different periods, both in the data and under optimal allocation. dy/y is the growth in labor productivity, measured in log differences, and $\Delta dy/y$ is the change in the growth compared to the previous period. Panel (a) and (c) present results from the value-added economy while panel (b) presents the results from the input-output economy.

Next, we examine how much of the slowdown in aggregate productivity growth can be attributed to the lack of improvement in allocation. We do so by comparing the productivity slowdown in the data to that under the optimal allocation. In essence, when we compute productivity growth under optimal allocation, we remove the contribution of allocation on growth, and the resulting growth rates can be taken as a reflection of the fundamentals. As shown in panel (a) of table 2, the magnitude of the productivity slowdown ($\Delta dy/y$) is significantly smaller under the optimal allocation than in the data. In the data, growth rates decline by 12 pp in the 1970s. Under the optimal allocation, however, we only see a decline of 3 pp. Similarly, during the 2000s, productivity growth slows down by 3 pp in the data, whereas it slows down by only 1 pp under the optimal allocation. Based on panels (b) and (c), the growth rate in the first half of the 2000s actually increases by 2 pp after removing the impact of allocation, and productivity growth only begins to stagnate in the second half.[8]

Figure 3: Capital and labor allocation



Notes: This figure plots the evolution of capital (E_k) and labor allocation (E_l) over time using measures from the value-added (VA) and the input-output (IO) economy.

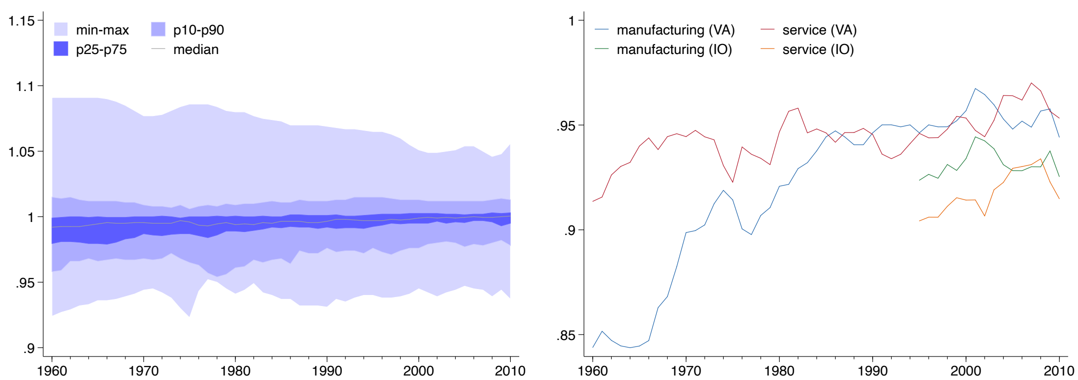
To better understand these findings, we perform a simple decomposition of $\mathbf{E_t}$ to see if it is driven primarily by capital or labor. More formally, in the value-added economy we define the measure of capital and labor allocation as $E^{k,t} = \prod_{i=1}^{N}(\frac{\chi_{i,t}^{k}}{\chi_{i,t}^{k*}})^{\alpha_{i,t}\theta_{i,t}}$ and $E^{l,t} = \prod_{i=1}^{N}(\frac{\chi_{i,t}^{l}}{\chi_{i,t}^{l*}})^{(1-\alpha_{i,t})\theta_{i,t}}$. In the input-output economy, the respective measures are $E^{k,t} =$

[8]One thing to note is that unlike the level of productivity, the growth rates of productivity can be lower under the optimal allocation than in the data. This can be seen in the table, which shows that in most decades, data growth rates are higher than the optimal allocation.

$\prod_{i=1}^{N}(((\frac{\chi^k_{i,t}}{\chi^{k*}_{i,t}})^{\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}})^{\sum_n \theta_{n,t} C_{ni,t}}$ and $E^{l,t} = \prod_{i=1}^{N}(((\frac{\chi^l_{i,t}}{\chi^{l*}_{i,t}})^{1-\alpha_{i,t}})^{1-\sigma_{i,t}-\lambda_{i,t}})^{\sum_n \theta_{n,t} C_{ni,t}}$. As shown in figure 3, throughout the sample period, capital allocation appears to be the more important driver. As with the long-run dynamics, capital allocation plays a more significant role during the two slowdown episodes as well. This pattern is evident in both value-added and input-output economies. However, since the data does not distinguish between capital returns and profits, we need to be cautious when interpreting this results especially in drawing policy implications. We provide more discussion of this point in section 5.2.

Figure 4: Sector-level allocative efficiency

(a) Full distribution **(b)** Manufacturing and service sector



Notes: Panel (a) plots the distribution of $E_{i,t}$ in a model without input-output linkages. The different shades of colors represent different percentiles of the $E_{i,t}$ distribution in year t. The grey line represents the median value of $E_{i,t}$. Panel (b) plots the allocative efficiency for the manufacturing and service sectors in both models. The classification of manufacturing and service sectors can be found in footnote 10.

Additionally, one might wonder if a particular sector is responsible for the overall movement. To see this, we decompose the aggregate measure into sectoral level allocative efficiency, such that $\mathbf{E}_t = \prod_{i=1}^{N} E_{i,t}^{\theta_{i,t}}$, where $E_{i,t} = (\frac{\chi^k_{i,t}}{\chi^{k*}_{i,t}})^{\alpha_{i,t}}(\frac{\chi^l_{i,t}}{\chi^{l*}_{i,t}})^{1-\alpha_{i,t}}$.[9] Intuitively, $E_{i,t} = 1$ means that sector i is at the optimal level while $E_{i,t} > 1$ and $E_{i,t} < 1$ imply that resources allocated to this sectors are higher or lower than the optimal allocation. In panel (a) of figure 4, we plot the distribution of $E_{i,t}^{\theta_{i,t}}$ where different shades of colors represent different percentiles of the distribution. This distribution would collapse into one point ($E_{i,t} = 1$

[9] We only present the results from the value-added economy in the main text, the results for the input-output model are similar.

for all i) under the optimal allocation, and a narrower distribution indicates a more efficient allocation. Based on this figure, there is a significant narrowing of the distribution from 1960 to 1970 and again from 1980 to 2000. In contrast, the distribution becomes more dispersed in the 1970s and remains relatively stable after 2000. That is, not surprisingly, the dynamics of this distribution is consistent with the movement of aggregate allocation efficiency. Further, the changes in $E_{i,t}$ can be seen at different percentiles of the distribution, which indicates that the aggregate dynamics is not driven by a single sector. In addition to the full distribution, we also plot the allocation of manufacturing and service sectors separately, such that $E_t^m = \prod_{i \in manufacturing} E_{i,t}^{\theta_{i,t}}$ and $E_t^s = \prod_{i \in service} E_{i,t}^{\theta_{i,t}}$.[10] As shown in panel (b), during the 1970, manufacturing and service sector both experience a decline in allocative efficiency, with service sector allocation deteriorating and recovering slightly earlier than the manufacturing sector. In contrast, the decline in allocative efficiency is relatively more concentrated in the manufacturing sectors in the 2000s.

To summarize, the exercises in this section show the quantitative importance of allocative efficiency. Moreover, we find that capital is the main driver of change, and the aggregate dynamics during slowdown episodes can be explained by sectors collectively moving away from their optimal level. In the following section, we provide evidence of an important force at work behind the evolution of allocative efficiency.

4.2 Volatility and allocation during the slowdown episodes

Existing work has shown that a higher level of time-series volatility is associated with deteriorations in allocation (Asker et al., 2014, Bloom et al., 2018 and Decker et al., 2020). The key mechanism can be illustrated using a canonical model of investment. In the presence of non-convex adjustment costs, there exists an inaction region where firms do not invest or disinvest. With an increase in volatility, this inaction region expands as the option value

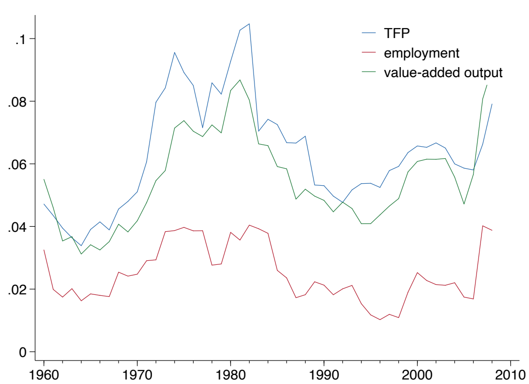
[10] As shown in table A.1, manufacturing sectors include all the sectors under the header "D manufacturing." Service sectors include the sectors under "G Wholesale and retail trade," "I Transportation and storage and communication," "K Real estate, renting and business activities", as well as sector H, J, M, N.

of waiting increases, which makes the aggregate allocation less efficient. These studies lead us to examine whether volatility is higher during the 1970s and 2000s. In the rest of the section, we document that these two decades are indeed characterized by higher volatility at the sector level, and provide evidence linking the increase in volatility to the deterioration in allocative efficiency.

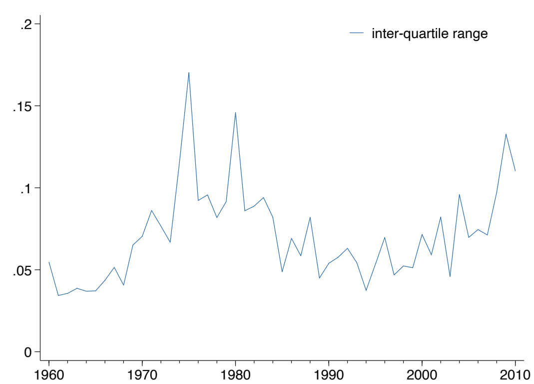
Productivity slowdown episodes are accompanied by high volatility For the first pass of the data, we compute sectoral growth volatility as the standard deviation of annual growth rates over a rolling 5-year window. The KLEMS dataset provides a range of sector-level outcomes. Here we report the volatility for employment, real value-added output, and TFP. As shown in panel (a) of figure 5, growth volatility starts to increase significantly at the beginning of the 1970s and remains elevated throughout the decade. At its highest point, the standard deviation is nearly double what it is in the 1960s. Volatility declines gradually between the 1980s and the beginning of the 2000s when it starts rising again. In other words, during the 1970s and 2000s, growth volatility is indeed significantly higher than the rest of the sample periods.

Figure 5: Sector-level shocks were more volatile during the 1970s and 2000s

(a) S.D. in sectoral growth rates



(b) Dispersion in TFP shocks



Notes: Panel (a) plots the cross-sectional s.d. of sectoral growth rates in employment, real value added output and TFP. Panel (b) plots the cross-sectional dispersion (inter-quartile range) in TFP shocks, computed as the residual terms from regression $\log A_{i,t} = \rho \log A_{i,t-1} + \mu_t + \chi_i + \epsilon_{i,t}$.

Next, we follow Bloom et al. (2018) to construct a measure of uncertainty. We calculate TFP shocks as the residuals ($\varepsilon_{i,t}$) from a regression equation for sector-level log TFP: $\log A_{i,t} = \rho \log A_{i,t-1} + \mu_t + \chi_i + \varepsilon_{i,t}$ and use the cross-sectional variance of the TFP shocks to measure uncertainty. While the standard deviation of growth rates reflects volatility in realized shocks, the dispersion in TFP shocks measures volatility un-forecasted by the regression equation. Panel (b) of figure 5 shows the inter-quartile range of $\varepsilon_{i,t}$ within each year. The 1970s and 2000 are again marked by greater volatility, as is the case with the previous measure of realized shocks.

These findings are in line with evidence at the establishment- or firm-level. Figure 3 of Bloom et al. (2018) documented the TFP shocks at the establishment level between 1972 and 2009, which followed a very similar pattern as figure 5.[11] Using the Census Bureau's Longitudinal Business Database (LBD) for 1981-2013, Decker et al. (2020) also showed that dispersion in TFP shocks was significantly higher in the 2000s than in the 1980s or 1990s. Collectively, the evidence documented here and in existing papers indicates that both productivity slowdown episodes are characterized by great volatility at the sectoral level as well as at the firm or establishment level.

Relationship between volatility and allocative efficiency Our next step is to conduct a more systematic evaluation of the relationship between volatility and allocative efficiency by exploiting variations in volatility over time. The results are shown in table 3. In column 1-3, we regress aggregate allocative efficiency $\log(\mathbf{E}_t)$ on cross-sectional dispersions of TFP shocks in year t, $t-1$, and $t-2$, while controlling for $\log(\mathbf{E}_{t-1})$. The inclusion of the past two years' measure of volatility is based on the insight that the history of TFP shocks might have a long-lasting impact on allocation in the presence of adjustment costs. The inclusion of $\log(\mathbf{E}_{t-1})$ aims at controlling for the long-run trend. The results confirm that higher volatility in year t is associated with a significantly less efficient allocation, with an

[11] Bloom et al. (2018) did not emphasize the larger TFP shocks in the 1970s and 2000s, as they focused on the impacts of uncertainty shocks at the business-cycle frequency.

estimated coefficient of -0.072 to -0.087. On the other hand, volatility of year $t-1$ and $t-2$ does not have a significant impact on year t's allocation, perhaps because their impacts are already captured by the allocation of $t-1$. In columns 4-6, we find very similar results when the dependent variables are the changes in allocative efficiency from $t-1$ to t. The estimated correlations range from -0.081 to -0.097 and remain highly significant.

Table 3: Relationship between volatility and allocative efficiency

	(1)	(2)	(3)	(4)	(5)	(6)
Dispersion of TFP shocks in year t	-0.072**	-0.087**	-0.080**	-0.081**	-0.097**	-0.095**
	(0.031)	(0.037)	(0.037)	(0.032)	(0.038)	(0.039)
Dispersion of TFP shocks in year $t-1$		0.003	0.009		0.005	0.007
		(0.043)	(0.046)		(0.041)	(0.046)
Dispersion of TFP shocks in year $t-2$			-0.006			0.002
			(0.027)			(0.030)
Allocative efficiency in year $t-1$	0.973***	0.981***	0.976***			
	(0.013)	(0.012)	(0.012)			
Dependent variables	$\log(\mathbf{E}_t)$	$\log(\mathbf{E}_t)$	$\log(\mathbf{E}_t)$	$\Delta\log(\mathbf{E}_t)$	$\Delta\log(\mathbf{E}_t)$	$\Delta\log(\mathbf{E}_t)$
N	63	62	61	63	62	61
R^2	0.993	0.993	0.993	0.093	0.137	0.128
Observed slowdown in 1970s	-0.12	-0.12	-0.12	-0.12	-0.12	-0.12
Predicted slowdown in 1970s	-0.10	-0.10	-0.10	-0.07	-0.08	-0.08
Observed slowdown in 2000s	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
Predicted slowdown in 2000s	-0.02	-0.03	-0.03	-0.03	-0.03	-0.03

Notes: The top half of the table presents the regression results. Column 1-3 regresses logarithms of $\mathbf{E_t}$ on the cross-sectional dispersion in TFP shocks from year $t, t-1, t-2$ while also controlling for the logarithms of $\mathbf{E_{t-1}}$. Column 4-6 regress the log difference in $\mathbf{E_t}$ on the cross-sectional dispersion in TFP shocks of year $t, t-1, t-2$. The bottom half of the table presents the predicted productivity slowdown using the estimated models. The predicted growth rates are computed as $\Delta \log \widehat{LP}_t = \Delta \log LP_t^* + \Delta \log \widehat{\mathbf{E}}_t$, where $\Delta \log LP_t^*$ are taken from previous estimates in table 2. Robust standard errors are reported in the parentheses.

We then ask to what extent volatility slows down productivity growth. To answer this question, we first obtain the predicted value for $\widehat{\mathbf{E}}_t$ using estimates from table 3, then calculate the predicted growth rates using the following equation: $\Delta \log \widehat{LP}_t = \Delta \log LP_t^* + \Delta \log \widehat{\mathbf{E}}_t$.[12] As shown at the bottom of table 3, the predicted slowdown during the 1970s ranges somewhere between 8 pp and 10 pp when we apply the full set of regressors (column 3

[12]For the regressions in column 4-6, we first obtain the predicted growth rates $\Delta \log \widehat{\mathbf{E}}_t$ and using the first year of $\mathbf{E_t}$ as a starting point, we compute $\widehat{\mathbf{E}}_t$ recursively for each year. The growth rate under the optimal allocation, $\Delta \log LP_t^*$, are taken from table 2.

and 6), accounting for more than two-thirds of the observed 12 pp slowdown in the data. The predicted slowdown during the 2000s is 3 pp, accounting for the entire observed slowdown in the data. According to table 2, $\Delta \log LP_t^*$ slows down by 3 pp during the 70s and 1 pp during the 00s, indicating that the remaining 5–7 pp and 2 pp, respectively, are predicted by the increase in volatility. Based on these findings, volatility plays an even greater role than fundamentals in driving the productivity slowdown.

Evidence at the sector level As of now, we have established the correlation between volatility and aggregate allocation efficiency. Similarly, allocative efficiency should also decline in sectors with more volatile growth. Based on these insights, we next provide evidence linking these two phenomena by exploiting cross-sector differences in growth volatility. More formally, for each sector i in year t, we compute a measure of volatility represented by the dispersion in TFP shocks over a rolling window from $t - \Delta t$ to t, as well as a measure for change in allocation represented by $|\log E_{i,t}| - |\log E_{i,t-\Delta t}|$.

Before discussing the results, it would be helpful to explain how we measure changes in allocative efficiency at the sector level. Recall that $E_{i,t}$ can be higher or lower than 1. Additionally, a decrease in $E_{i,t}$ does not necessarily mean worse allocation. For example, a change from $E_{i,t} = 1.2$ to $E_{i,t} = 0.9$ actually indicates that the allocation is more efficient. This is because the size of the gap between $E_{i,t}$ and 1, which represents how far this sector is from the optimal allocation, decreases from 0.2 to 0.1. Therefore, the proper measure for sectoral allocation efficiency is the absolute value of the difference between $E_{i,t}$ and 1, which can be written in log terms as $|\log E_{i,t}|$. Consequently, the change in allocation is measured by $|\log E_{i,t}| - |\log E_{i,t-\Delta t}|$, where a positive value indicates a deterioration in allocation over the window $[t - \Delta t, t]$.[13]

The estimated results in table 4 show that an increase in the dispersion of TFP shocks is associated with a significant deterioration of allocation at the sector level.[14] With a rolling

[13] Or we could measure the change in allocation as $|\log E_{i,t-\Delta t}| - |\log E_{i,t}|$, where a positive value indicates an improvement in allocation.

[14] The dispersion in TFP shocks is measured as the inter-quartile range in this table. The results are

Table 4: Sector-level relationship between volatility and allocative efficiency

Dependent variable $\lvert\log E_{i,t}\rvert - \lvert\log E_{i,t-\Delta t}\rvert$		$\Delta t = 2$			$\Delta t = 4$	
Dispersion of TFP shock in $[t-\Delta t, t]$	0.141**	0.168*	0.154*	0.235**	0.253**	0.224*
	(0.0713)	(0.0866)	(0.0925)	(0.101)	(0.124)	(0.134)
Sector FEs	N	Y	Y	N	Y	Y
Year FEs	N	N	Y	N	N	Y
N	1593	1593	1593	1593	1593	1593
R^2	0.021	0.060	0.104	0.029	0.081	0.127
		$\Delta t = 6$			$\Delta t = 8$	
Dispersion of TFP shock in $[t-\Delta t, t]$	0.317***	0.437***	0.396***	0.541***	0.722***	0.671***
	(0.0895)	(0.120)	(0.134)	(0.123)	(0.173)	(0.191)
Sector FEs	N	Y	Y	N	Y	Y
Year FEs	N	N	Y	N	N	Y
N	1593	1593	1593	1593	1593	1593
R^2	0.056	0.139	0.188	0.070	0.181	0.231

Notes: This table regresses changes in allocative efficiency over $[t-\Delta t, t]$ on growth volatility—measured by the dispersion of TFP shocks—at the sector level. The regressions with or without sector- and year-fixed effects are presented in different columns. In addition, we also show the regression results with rolling windows of different lengths, where $\Delta t = 2$, $\Delta t = 4$, $\Delta t = 6$, and $\Delta t = 8$ correspond to 3-year, 5-year, 7-year and 9-year rolling windows. Robust standard errors are reported in the parentheses.

window of three years ($\Delta t = 2$), an increase in sectoral volatility is associated with a 0.14 pp decline in allocative efficiency. The point estimates increase somewhat after controlling for one or both sets of the sector- and year-fixed effects and remain significant. We run the regression using rolling windows of various lengths. The estimated relationship is stronger and more significant when extending the length of the rolling window.[15] Overall, we find a robust and significant correlation between higher volatility and low allocative efficiency, consistent with what we see at the aggregate level.

Additionally, we find that the direction of $E_{i,t}$'s movement is correlated with the sign of the TFP shocks. In theory, the optimal level of resources allocated to a sector receiving a positive shock should increase. Due to the adjustment costs, however, on average the amount of resources flowing to this sector would be less than optimal. Consequently, with

similar when we use the s.d. of TFP shocks.

[15] We present the results for a 3-year, 5-year, 7-year, and 9-year rolling window. They correspond to $\Delta t = 2$, $\Delta t = 4$, $\Delta t = 6$, and $\Delta t = 8$ in the table.

Table 5: Direction of movements in $E_{i,t}$ is correlated with the sign of TFP shocks

Dependent variable $\mathbb{I}(E_{i,t} < E_{i,t-\Delta t})$	$\Delta t = 2$	$\Delta t = 4$	$\Delta t = 6$	$\Delta t = 8$
Dummy indicator of positive accumulative TFP shocks	0.204***	0.214***	0.119***	0.116***
	(0.0251)	(0.0248)	(0.0247)	(0.0250)
N	1593	1593	1593	1593
R^2	0.170	0.196	0.205	0.231
Dummy indicator of positive median TFP shocks	0.195***	0.162***	0.0997***	0.0882***
	(0.0250)	(0.0247)	(0.0240)	(0.0238)
N	1593	1593	1593	1593
R^2	0.167	0.179	0.202	0.227
TFP shocks (accumulative)	0.258***	0.234***	0.133***	0.137***
	(0.0765)	(0.0691)	(0.0492)	(0.0408)
N	1593	1593	1593	1593
R^2	0.110	0.137	0.175	0.210
TFP shocks (median)	0.886***	0.923***	0.591*	0.820**
	(0.305)	(0.288)	(0.316)	(0.367)
N	1593	1593	1593	1593
R^2	0.106	0.133	0.172	0.206

Notes: This table examines the correlation between the sign of TFP shocks and the direction of the movement in $E_{i,t}$. The left-hand-side variable is a dummy variable indicating if there is a decline in $E_{i,t}$ over $[t - \Delta t, t]$. On the right-hand side, we include a dummy variable indicating a positive accumulative or median TFP shock or the actual values of the shocks. All regressions include a set of the sector- and year-fixed effects. Robust standard errors are reported in the parentheses.

a positive TFP shock, we are likely to witness a decrease in $E_{i,t}$ as the data allocation falls behind the optimal level. We test this prediction in table 5. To do so, we first construct a dummy variable $\mathbb{I}(E_{i,t} < E_{i,t-\Delta t})$ indicating if there is a decrease in $E_{i,t}$ from $t - \Delta t$ to t. We then regress this variable on a dummy variable that indicates the sector experiences positive shocks over the period $[t - \Delta t, t]$ while controlling for the sector- and year-fixed effects. We expect the estimated coefficient to be positive and significant. As shown in the top panel of table 5, this is indeed what the data shows.[16] The estimated correlation is 0.2 using a three-year rolling window and somewhat lower with longer rolling windows but

[16] The accumulative TFP shock is defined as the sum of TFP shocks $\epsilon_{i,t}$ over the period $[t - \Delta t, t]$.

remains highly significant. When we consider the median value of the TFP shocks instead of the accumulative value, we find very similar results, despite slightly lower point estimates. Finally, we replace the dummy variables for the positive TFP shocks with the actual values in the regressions. As shown in the bottom two panels, a more positive TFP shock is associated with a higher probability of a decrease in $E_{i,t}$.[17]

In sum, the two exercises in this section provide insight into the movement of each $E_{i,t}$ as the cross-sectional distribution of $E_{i,t}$ determines the aggregate allocative efficiency. The magnitude, as well as the direction of the changes in $E_{i,t}$, are closely linked to the underlying TFP shocks and their relationships are in line with the predictions from the model.

Evidence from factor utilization Next, we present additional evidence supporting the findings based on the dynamics of factor utilization rates. Empirically, recent studies documented that the dispersion of capacity utilization rates is an important contributor to the dispersion of marginal product of capital and labor across firms (Gorodnichenko et al., 2021).[18] Using a model of investment with adjustment costs, Abel and Eberly (1998) showed that firms' position in the inaction region is represented by their factor utilization rates. As volatility increases, the inaction region expands, resulting in a wider range of utilization rates in the cross-section. Consequently, the two episodes of productivity slowdown should be accompanied by increased dispersion of utilization rates, which we now investigate next.

To do so, we first construct measures of factor utilization rates. For capital, a common approach uses energy consumption to calculate utilization-adjusted capital service (Burnside et al., 1995). According to the estimation in Burnside et al. (1995), actual capital service used, $\hat{K}_{i,t}$, can be written as a Leontief function of capital stock and energy consumption. Using the specification in Oberfield (2013), we let $\hat{K}_{i,t} = min\{E_{i,t}/b_i, u_{i,t}K_{i,t}\}$, where $u_{i,t}$ is the capital utilization rate, $E_{i,t}$ is the energy consumption, and b_i represents the energy

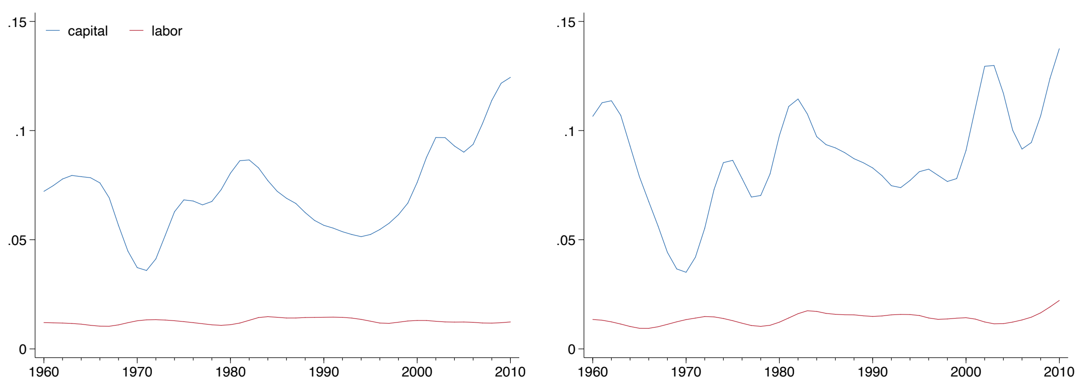
[17] The same pattern holds when replacing the left-hand-side dummy variable with the actual changes in $E_{i,t}$.

[18] Capacity and factor utilization rates are different but related concepts, where the former is a measure based on output and the latter a measure based on inputs (see Abel and Eberly, 1998 for discussions).

intensity of capital usage in each sector determined by its fundamental technology. The difference between the growth rates of $\hat{K}_{i,t}$ and $K_{i,t}$ therefore represents the changes in capital utilization rates: $\Delta \log u^k_{i,t} = \Delta \log \hat{K}_{i,t} - \Delta \log K_{i,t}$. For labor utilization rates, we follow the tradition of the growth accounting literature and define it as the average hours worked per employed person (Basu et al., 2006 and Fernald, 2014a). This then allow us to compute the changes in the labor utilization rate $\Delta \log u^l_{i,t}$ for each sector on a yearly basis as well. The cross-sectional dispersion in $\Delta \log u^k_{i,t}$ and $\Delta \log u^l_{i,t}$ thus is used to measure the dispersion in factor utilization rates in the cross section.

Figure 6: Cross-sectional dispersions of factor utilization rates

(a) Standard deviation **(b)** Inter-quartile range



Notes: This table plots the cross-sectional dispersions in capital and labor utilization rates every year. Panel (a) and (b) plot the standard deviation and interquartile range, respectively. The time series are HP-filtered with a smoothing parameter of 6.25.

Figure 6 displays the HP-filtered time-series of the s.d. (panel a) and inter-quartile range (panel b) of the utilization rates on a yearly basis. Labor utilization dispersion is relatively smaller and stable over time, compared to capital utilization dispersion, which varies greatly over time. Notably, during the 1970s and 2000s, the dispersion of capital utilization rates increase significantly compared to the previous decades, in line with the model predictions. Despite being suggestive, this figure provides further evidence that higher volatility is likely an important determinant of changes in allocative efficiency.[19]

[19] A caveat to this analysis is that the data only allows us to calculate the changes in utilization rates for

The above analysis focuses on the dispersion of utilization rates across sectors. Suppose instead there are variations in the aggregate utilization rate over time. As long as the variations are uniform across sectors, they should not impact the cross-sector allocation directly. However, unmeasured fluctuations in aggregate utilization rate could bias the measurement of aggregate productivity. Adjustment of utilization generally attenuates aggregate productivity fluctuations, but the two episodes of productivity slowdown persist after the adjustment (Fernald, 2014b and Comin et al., 2020). To the extent that the productivity slowdown is less severe after adjusting for utilization and the measured allocation not affected, we have likely understated the explanatory power of allocative efficiency.

5 Robustness exercises

This section provides robustness checks of the main results. We consider (i) an extension of the benchmark Cobb-Douglas framework to the CES production system, (ii) different measures for capital and labor inputs, (iii) non-zero profits, and (iv) alternative specifications to estimate the output elasticities.

5.1 CES production system

Our baseline results are based on the Cobb-Douglas production system, which serves as a valuable benchmark. Recent papers have shown that the loss from misallocation depends on the elasticity of substitution between inputs (see Epifani and Gancia, 2011 and Osotimehin and Popov, 2020). One main takeaway from these papers is that the loss from misallocation is lower with a lower elasticity of substitution between inputs (higher complementarity).

In this section, we extend the benchmark value-added economy to a more flexible CES production system. More formally, the final good is a CES aggregation of the intermediate goods, such that $Y = (\sum_i \omega_i Y_i^{1-\frac{1}{\rho}})^{\frac{\rho}{\rho-1}}$. The intermediate good Y_i is produced using capital

each year but not the levels. If the level of the utilization rates were available, we could quantify to what extent utilization rates contribute to the cross-sectional variations in $E_{i,t}$ as in Gorodnichenko et al. (2021).

and labor $Y_i = A_i(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}$. In these production functions, $(\omega_i,\ \nu_i)$ are the CES weights and (ρ, ϵ) are the elasticity of substitution. Similar as before, the planner solves the following optimization problem: $max\ Y, s.t\ \sum_i K_i = K, \sum_i L_i = L$. Appendix section E provides details of the model solution.

To evaluate our results in the CES framework, we first estimate the elasticity of substitution parameters—ρ and ϵ—using the following specifications

$$\log \frac{p_{i,t}Y_{i,t}}{p_{N,t}Y_{N,t}} = \beta_\rho \log \frac{Y_{i,t}}{Y_{N,t}} + \chi_i + u_{i,t}, \tag{7}$$

$$\log \frac{R_i K_i}{w_i L_i} = \beta_\epsilon \log \frac{K_i}{L_i} + \chi_i + u_{i,t}, \tag{8}$$

where $\beta_\rho = \frac{\rho}{\rho-1}$, $\beta_\epsilon = \frac{\epsilon}{\epsilon-1}$, χ_i represents the sector fixed effects and $u_{i,t}$ is the error term.[20] In equation 7, $p_{i,t}Y_{i,t}$ is the nominal value-added output for sector $i \in \{1, ..., N\}$ and $Y_{i,t}$ is the corresponding real value-added output. In equation 8, $R_i K_i$ and $w_i L_i$ are the capital and labor income in nominal terms while K_i and L_i are the real capital stock and number of workers in each sector. Intuitively speaking, the identification strategy exploits the relationship between changes in inputs expenditure and changes in inputs quantity over time.[21] After obtaining the elasticity of substitution parameters, the CES weights in the production function can then be calculated using the expenditure shares over a rolling window as in the Cobb-Douglas case.

The point estimate for the elasticity of substitution between sector goods is $\rho = 0.96$, smaller than but not significantly different from 1. The elasticity of substitution ρ measures how easy it is for consumers to substitute across a broad set of goods or services. Not surprisingly, the estimates would vary somewhat across different sectoral classification schemes. In Aum et al. (2018), the authors categorized all non-agriculture industries into ten

[20] Both specifications are derived from the cost-minimization conditions following the approach in Aum et al. (2018). The conditions that generate equation 7 and 8 are $\log(\frac{P_i Y_i}{P_N Y_N}) = \log(\frac{\omega_i}{\omega_N}) + \frac{\rho-1}{\rho}\log(\frac{Y_i}{Y_N})$ and $\log \frac{R_i K_i}{w_i L_i} = \log \frac{\lambda_i}{1-\lambda_i} + \frac{\epsilon-1}{\epsilon}\log(\frac{K_i}{L_i})$, respectively.

[21] Given the expenditure of inputs, price and quantity of the inputs provide essentially the same information. Therefore, the above specification is equivalent to one that regresses expenditure on prices (Atalay, 2017) or regresses quantity on prices (Oberfield and Raval, 2021).

broad sectors. Their estimated elasticity was 0.77 across these sectors. Oberfield and Raval (2021) showed that the estimates of elasticity across two-digit manufacturing sectors were centered around one from various specifications. In Herrendorf et al. (2013), the benchmark specification estimated that the elasticity between broad sectors—agriculture, service, and manufacturing—to be around 0.9. Atalay (2017) suggested that a value smaller but closer to one best characterizes the demand elasticity from consumers. Both Atalay (2017) and Oberfield and Raval (2021) chose elasticity equal to one in their baseline parametrization. Overall, we find that our estimate is within the range of the estimates in the literature.

Our estimated elasticity of substitution between capital and labor is $\epsilon = 0.81$, suggesting that capital and labor are gross complements in the sectoral production functions. Several recent papers also estimated the elasticity of substitution between capital and labor at the sectoral/industry level. Among them, Herrendorf et al. (2015) considered three broad sectors—agriculture, manufacturing, and service, and the estimated elasticity of substitution between capital and labor are 1.58, 0.8, and 0.75 in these three sectors, respectively. Alvarez-Cuadrado et al. (2017) found a slightly lower value for the manufacturing (0.78) and service sector (0.57). By aggregating up elasticities at the plant level, Oberfield and Raval (2021) obtained an elasticity of 0.72 for manufacturing sectors in 1987 and suggested that it has been trending down since. In Aum et al. (2018), the authors combined labor with two types of capital—computer and non-computer—using a nested CES structure, and found that the estimated elasticity between computer capital and labor ranged from 1.2 to 1.8.[22]

Given the empirical challenges associated with estimating the elasticity of substitution and the fact that the literature has yet to achieve a consensus, it would be necessary to apply a range of values for these parameters. We set our baseline specification to $\rho = 0.96$

[22] Researchers have also estimated the elasticity of substitution between capital and labor at the aggregate level. There exists a relatively wide range of estimates (see Chirinko, 2008 for a summary) in the literature. For example, in Karabarbounis and Neiman (2014), the authors estimated that capital and labor are gross substitutes with an estimated elasticity between 1.2 and 1.5 using several cross-country aggregate datasets (see also the estimates in Piketty, 2017). On the other hand, most other papers have found that capital and labor are gross complements, including Antràs (2004), Klump et al. (2007), and León-Ledesma et al. (2010), among others.

and $\epsilon = 0.81$, which are directly estimated using our data. We also consider the case of a lower value for the elasticity between sectoral goods, $\rho = 0.77$ (Aum et al., 2018). For the elasticity between capital and labor, we take estimated value from Oberfield and Raval (2021) (0.72) and Karabarbounis and Neiman (2014) (1.25), which reflect the different views of whether capital and labor are gross substitutes or complements. This gives us in total six combinations of parameter values for ρ and ϵ.

Table 6: Changes in productivity growth ($\Delta \frac{dy}{y}$) under optimal allocation

	CD	CES					
		$\rho = 0.96$			$\rho = 0.77$		
		$\epsilon = 0.81$	$\epsilon = 0.72$	$\epsilon = 1.25$	$\epsilon = 0.81$	$\epsilon = 0.72$	$\epsilon = 1.25$
		baseline		highest elas.		lowest elas.	
1960-69	–	–	–	–	–	–	–
1970-79	-0.03	-0.04	-0.05	-0.02	-0.05	-0.06	-0.04
1980-89	-0.03	-0.02	-0.01	-0.04	-0.02	0.00	-0.03
1990-99	0.06	0.06	0.06	0.05	0.06	0.06	0.05
2000-07	-0.01	-0.01	-0.01	-0.00	-0.02	-0.01	-0.00

Notes: This table presents the change in productivity growth ($\Delta dy/y$) compared to the previous decade under the optimal allocation in the CES framework. The results are presented under six combinations of parameter values for ρ and ϵ, the elasticity of substitution parameters in the production system. The baseline parameterization is $\rho = 0.96$ and $\epsilon = 0.81$. As a comparison, we also include the results from the Cobb-Douglas case, taken from the last column of table 2.

Table 6 presents the changes in productivity growth compared to the previous decade under the optimal allocation.[23] With the baseline parameterization ($\rho = 0.96, \epsilon = 0.81$), productivity growth in the 1970s and 2000s slows down by 4 pp and 1pp. We note that the difference between the baseline CES result and the Cobb-Douglas case is rather small. Further, we find that higher elasticity is generally associated with a more prominent role for allocation to explaining the productivity slowdown, consistent with the findings in Epifani and Gancia (2011) and Osotimehin and Popov (2020).[24] For the case with the highest elasticity of substitution ($\rho = 0.96, \epsilon = 1.25$), productivity slowdown disappears in the 2000s and the growth rate is only lowered by 2 pp during the 1970s. Even for the scenario

[23] In appendix section B.1, we provide more details including the evolution of allocative efficiency under the different values of elasticity over the whole sample period.

[24] Epifani and Gancia (2011) and Osotimehin and Popov (2020) focused on the level of allocative efficiency while here we focus on the growth rates.

with the lowest elasticity ($\rho = 0.77, \epsilon = 0.72$), allocation can explain at least half of the productivity slowdown. Overall, our results confirm the empirical importance of the elasticity of substitution. Under a range of empirically plausible parameter values, allocative efficiency explains a significant portion of the productivity slowdown.[25]

5.2 Additional robustness checks

In this section, we discuss several additional robustness checks. To avoid repetition, for each exercise, we provide a summary of the results in the main text and include more details in the appendix section B.

Different measure for capital and labor inputs So far our empirical analysis relies on the 2013 version of KLEMS and WOIT. Although the construction of KLEMS data takes into account the different composition of capital types in each sector, one might wonder if this aggregation would cause mismeasurement.[26] The same concern applies to the measurement of labor too, which is usually addressed in literature by using wage bills as a proxy for labor inputs.

In this section, we repeat the exercise using alternative input measures. We take capital inputs by different types from the 2009 version of KLEMS, which covers a shorter period (1977-2007). The data reports eight asset types: (i) computing equipment, (ii) communications equipment, (iii) software, (iv) transport equipment, (v) other machinery and equipment, (vi) total non-residential investment, (vii) residential structures, and (viii) other assets. The first three asset types constitute the ICT asset group and the latter five constitute the non-ICT group. For labor inputs, we follow the literature to use labor compensation as a proxy.[27]

[25] Our exercise abstracts from the complementarity between intermediate inputs. With the availability of better data, the framework can be extended to incorporate this dimension, thereby potentially enhancing the impact of complementarity (see models in Atalay, 2017 and Osotimehin and Popov, 2020).

[26] See Jorgenson et al. (2014) for details on how the industry-level capital data are constructed.

[27] These alternative measures, however, come with their own set of challenges. In constructing the returns to capital by type, assumptions are needed to split the total capital income into returns of different types. This may introduce measurement errors. When it comes to labor inputs, wage bills may address the composition problem, but they cannot distinguish the role of price and quantity in measuring allocation.

Details of the exercises can be found in the appendix section B.2.

When extending the analysis to two types of assets—ICT and non-ICT capital, the result is only available from the end of the 1970s. We find a rapid improvement in allocation during the 80s and a gradual deterioration since the beginning of the 2000s. Similar trends are found in the exercise with eight types of assets. Under optimal allocation, productivity slows down by 1 pp and 2 pp in the 2000s in these two exercises. Additionally, by replacing employment with wage bills as a measure of labor input, the level of allocative efficiency is slightly higher, but the trend remains very similar to the baseline result. In this case, the slowdown in productivity completely vanishes in the 2000s and shrinks by half during the 1970s. In sum, with these alternative measures, allocation explains at least half of the slowdown in the 1970s and one-third of the slowdown in the 2000s.

Profits In the KLEMS dataset, capital income is constructed as value-added minus labor income; in other words, the underlying assumption is that sectors make zero pure profits. This data treatment is partly based on findings that pure profits as a share of value-added output are close to zero in the US (see for example Rotemberg and Woodford, 1995). However, recent studies showed that profits shares have been rising in the US. This implies that the data might have overestimated capital income; consequently, this would lead to an upward bias of the capital-output elasticity in the production functions and an overstatement of capital weights in measuring allocative efficiency.

We next relax the assumption of zero pure profits. We first split the raw capital compensation into pure profits and actual capital income using the estimated profits to capital income ratio by Barkai (2020). There exists a range of estimates for this ratio over time. We choose two values that are on the upper end of the range: (i) ratio=1/5 in 2010, the last year of our sample, and (ii) ratio=1/2 in 2007, the highest level over the sample period in Barkai (2020). Appendix section B.3 includes details of the exercise. Here we provide a summary of the main take-aways. We find that under the assumption of positive profits, the changes in allocative efficiency are of a slightly smaller magnitude, which perhaps is an indication of the presence of bias in the baseline result. However, our main finding is robust under this alternative assumption. Even with the highest profit share, allocation stills explain at least half of the productivity slowdown in the data.

In the analysis above, we relax the assumption of zero pure profits but assume that profit shares are the same across sectors. Suppose instead markups are heterogeneous across sectors. This would cause dispersions in profits and misallocation of resources (Epifani and Gancia, 2011 and Baqaee and Farhi, 2020). In our exercise, we attribute those dispersions to capital income. On the other hand, if we had good quality data for sector-level profits, we could separate the dispersion of markups and capital income. This will then allow us to draw better policy conclusions. At the same time, it means that we should be cautious in interpreting the current results, since part of capital "misallocation" may be caused by variations in markups across sectors.

Alternative specifications The output elasticity in the production functions is an important set of parameters for our empirical exercises. These parameters are closely linked to the factor expenditure shares in the data. However, these expenditure shares in the data might be distorted, therefore we conduct our analysis using two different specifications. In the main text, we report the results based on the first specification and 3-year rolling window. In this paragraph, we discuss the results from the other specifications. Details of the exercises can be found in appendix sections B.4 and B.5. Below we summarize the main findings.

First, we increase the length of the rolling window within which we calculate the average expenditure shares. The idea behind this is that the average expenditure shares are more likely to be undistorted over a longer window as a result of the law of large numbers. We find that with longer rolling windows, the evolution of allocative efficiency is slightly smoother than the baseline, but the resulting implications for the productivity slowdown are virtually unchanged. Second, we assume that the expenditure shares in the later years are undistorted and apply them to the earlier years of the sample. Specifically, in the exercise, we choose two base years: (i) 2010, the last year of our sample, and (ii) 2005, to avoid the impact of the Great Recession. Under this specification, we find that the deterioration in allocation is less prominent than the baseline results during the 70s. However, the role of allocation remains quantitatively significant as it still explains more than half of the productivity slowdown. In sum, we find that our results are not sensitive to the specification used in the baseline results. Rather, we document similar findings using a variety of alternative specifications.

6 Conclusion

In this paper, we quantify how much of the slowdown in productivity growth can be explained by factor allocation. We apply a tractable decomposition framework to the US economy and show that allocative efficiency explains approximately two-thirds of the productivity slowdown in both the 1970s and 2000s. A variety of factors might have caused the deviation from optimal allocation in the data. Providing additional evidence on the role of volatility, we find that the worsening allocation in both slowdown episodes is partly due to the increased volatility.

These findings demonstrate the important role of volatility in affecting allocation and aggregate productivity growth. In theory, the worsening allocation in highly volatile times can stem from firms optimizing their production in the presence of adjustment costs. In this case, a departure from the unconstrained optimal allocation (without adjustment costs) might not be constrained inefficient; hence policies that directly target firms' decisions are not necessarily welfare-enhancing. In contrast, policy initiatives that reduce the volatility faced by firms or lower their adjustment costs are undoubtedly conducive to growth. As economies emerge from the COVID crisis, these policies will likely be even more crucial as their path to full recovery remains very uncertain.

We hope that this framework may prove useful in studying other issues related to allocation and growth. One particular direction of work comes to mind. In recent decades, several countries have experienced fast growth and significant catching up to the frontier, and the improvement in allocation efficiency has played an important role in this process (Buera and Shin, 2013 and Song et al., 2011). In general, however, there is significant heterogeneity in convergence patterns across countries, even within the group of advanced economies (see for example Cette et al., 2016). On average, developing countries as a whole have not made much progress in closing the income gaps (Johnson and Papageorgiou, 2020). This leads to the question of how much of these patterns are explained by variations in allocation efficiency across countries. An extension of the current framework to the cross-country setting could provide some insights on this question.

References

Abel, A. B. and J. C. Eberly (1998): "The mix and scale of factors with irreversibility and fixed costs of investment," *Carnegie-Rochester Conference Series on Public Policy*, 48, 137–144. 27

Alvarez-Cuadrado, F., N. Long, and M. Poschke (2017): "Capital-labor substitution, structural change and growth," *Theoretical Economics*, 12. 31

Antràs, P. (2004): "Is the U.S. aggregate production function Cobb-Douglas? New estimates of the elasticity of substitution," *Contributions to Macroeconomics*, 4. 31

Asker, J., A. Collard-Wexler, and J. D. Loecker (2014): "Dynamic inputs and resource (Mis)allocation," *Journal of Political Economy*, 122, 1013–1063. 3, 20

Atalay, E. (2017): "How Important Are Sectoral Shocks?" *American Economic Journal: Macroeconomics*, 9, 254–280. 30, 31, 33

Aum, S., S. Y. T. Lee, and Y. Shin (2018): "Computerizing Industries and Routinizing Jobs: Explaining Trends in Aggregate Productivity," *Journal of Monetary Economics*, 97, 1–21. 5, 30, 31, 32, 2

Baily, M. N. and R. J. Gordon (1988): "The Productivity Slowdown, Measurement Issues, and the Explosion of Computer Power," *Brookings Papers on Economic Activity*, 19, 347–432. 4

Baqaee, D. R. and E. Farhi (2020): "Productivity and Misallocation in General Equilibrium," *Quarterly Journal of Economics*, 135, 105–163. 6, 11, 35

Barkai, S. (2020): "Declining Labor and Capital Shares," *Journal of Finance*, 75, 2421–2463. 4, 34, 5, 6

Basu, S. and J. G. Fernald (2002): "Aggregate Productivity and Aggregate Technology," *European Economic Review*, 46, 963–991. 6

Basu, S., J. G. Fernald, and M. S. Kimball (2006): "Are technology improvements contractionary?" *American Economic Review*, 96, 1418–1448. 28

Baumol, W. J. (1967): "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis," *The American Economic Review*, 57, 415–426. 5

Bils, M., P. J. Klenow, and C. Ruane (2020): "Misallocation or Mismeasurement?" Working paper. 14

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2018): "Really Uncertain Business Cycles," *Econometrica*, 86, 1031–1065. 3, 20, 22

Bloom, N., C. I. Jones, J. Van Reenen, and M. Webb (2020): "Are Ideas Getting Harder to Find?" *American Economic Review*, 110, 1104–44. 4

Buera, F. J. and Y. Shin (2013): "Financial Frictions and the Persistence of History: A Quantitative Exploration," *Journal of Political Economy*, 121, 221–272. 37

Burnside, C., M. Eichenbaum, and S. Rebelo (1995): *Capital Utilization and Returns to Scale*, vol. 10. 27

Byrne, D. M., J. G. Fernald, and M. B. Reinsdorf (2016): "Does the United States Have a Productivity Slowdown or a Measurement Problem?" *Brookings Papers on Economic Activity*, 2016, 109–182. 4

Calligaris, S., M. D. Gatto, F. Hassan, G. Ottaviano, and F. Schivardi (2018): "The Productivity Puzzle and Misallocation: An Italian Perspective," Working paper. 6

Cette, G., J. Fernald, and B. Mojon (2016): "The Pre-Great Recession Slowdown in Productivity," *European Economic Review*, 88, 3–20. 4, 37

Chirinko, R. S. (2008): "σ: The long and short of it," *Journal of Macroeconomics*, 30, 671–686. 31

Comin, D. A., J. Q. Gonzalez, T. G. Schmitz, and A. Trigari (2020): "Measuring TFP: the role of profits, adjustment costs, and capacity utilization," NBER Working Paper. 29

Davis, S. J. and J. Haltiwanger (2001): "Sectoral job creation and destruction responses to oil price changes," *Journal of Monetary Economics*, 48, 465–512. 5

Decker, R. A., J. Haltiwanger, R. S. Jarmin, and J. Miranda (2020): "Changing Business Dynamism and Productivity: Shocks versus Responsiveness," *American Economic Review*, 110, 3952–3990. 3, 4, 5, 20, 22

Duernecker, G., B. Herrendorf, and A. Valentinyi (2017): "Structural Change within the Service Sector and the Future of Baumol's Disease," . 5

Epifani, P. and G. Gancia (2011): "Trade, Markup Heterogeneity and Misallocations," *Journal of International Economics*, 83, 1–13. 29, 32, 35, 2

Fernald, J. G. (2014a): "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity," *Federal Reserve Bank of San Francisco, Working Paper Series*, 01–28. 28

——— (2014b): *Productivity and potential output before, during, and after the great recession*, vol. 29. 29

Feyrer, J. (2007): "Demographics and Productivity," *The Review of Economics and Statistics*, 89, 100–109. 4

Gopinath, G., Ş. Kalemli-Özcan, L. Karabarbounis, and C. Villegas-Sanchez (2017): "Capital Allocation and Productivity in South Europe," *The Quarterly Journal of Economics*, 132, 1915–1967. 6

Gordon, R. J. (2016): *The Rise and Fall of American Growth: The U.S. Standard of Living since the Civil War*, Princeton University Press, 1 ed. 4

Gorodnichenko, Y., D. Revoltella, J. Svejnar, and C. Weiss (2021): "Resource Misallocation in European Firms: The Role of Constraints, Firm Characteristics and Managerial Decisions," Working Paper. 27, 29

Greenwood, J. and M. Yorukoglu (1997): "1974," *Carnegie-Rochester Conference Series on Public Policy*, 46, 49–95. 4

Herrendorf, B., C. Herrington, and A. Valentinyi (2015): "Sectoral Technology and Structural Transformation," *American Economic Journal: Macroeconomics*, 7, 104–33. 31

Herrendorf, B., R. Rogerson, and A. Valentinyi (2013): "Two Perspectives on Preferences and Structural Transformation," *American Economic Review*, 103, 2752–2789. 31

Hornstein, A. and P. Krusell (1996): "Can Technology Improvements Cause Productivity Slowdowns?" in *NBER Macroeconomics Annual 1996, Volume 11*, National Bureau of Economic Research, Inc, NBER Chapters, 209–276. 4

Hsieh, C.-T. and P. J. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 124, 1403–1448. 5

Johnson, P. and C. Papageorgiou (2020): "What Remains of Cross-Country Convergence?" *Journal of Economic Literature*, 58, 129–75. 37

Jones, C. I. (2013): *Misallocation, Economic Growth, and Input-Output Economics*, Cambridge University Press, vol. 2 of *Econometric Society Monographs.*, 419–456. 1, 5

Jorgenson, D. W. (1988): "Productivity and Postwar U.S. Economic Growth," *Journal of Economic Perspectives*, 2, 23–41. 4

Jorgenson, D. W., M. S. Ho, A. Patel, and J. D. Samuels (2014): "Data Appendix to 'Economic Growth in the Information Age'," . 33, 3

Karabarbounis, L. and B. Neiman (2014): "The Global Decline of the Labor Share," *Quarterly Journal of Economics*, 129, 61–103. 31, 32, 2

Klump, R., P. McAdam, and A. Willman (2007): "Factor Substitution and Factor-Augmenting Technical Progress in the United States: A Normalized Supply-Side System Approach," *The Review of Economics and Statistics*, 89, 183–192. 31

León-Ledesma, M. A., P. McAdam, and A. Willman (2010): "Identifying the elasticity of substitution with biased technical change," *American Economic Review*, 100, 1330–1357. 31

Oberfield, E. (2013): "Productivity and Misallocation During a Crisis: Evidence from the Chilean Crisis of 1982," *Review of Economic Dynamics*, 16, 100–119. 1, 6, 14, 27

Oberfield, E. and D. Raval (2021): "Micro Data and Macro Technology," *Econometrica*, 89, 703–732. 30, 31, 32, 2

Osotimehin, S. and L. Popov (2020): "Misallocation and Intersectoral Linkages," Working Paper. 29, 32, 33, 2

Piketty, T. (2017): *Capital in the Twenty-First Century*, Harvard University Press. 31

Ramey, V. A. (2020): "Secular Stagnation or Technological Lull?" *Journal of Policy Modeling*, 42, 767–777. 4

Rotemberg, J. J. and M. Woodford (1995): "Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets," in *Frontiers of Business Cycle Research*, ed. by T. Cooley, Princeton University Press, 243–293. 34

Song, Z., K. Storesletten, and F. Zilibotti (2011): "Growing Like China," *American Economic Review*, 101, 196–233. 37

Syverson, C. (2017): "Challenges to Mismeasurement Explanations for the US Productivity Slowdown," *Journal of Economic Perspectives*, 31, 165–86. 4

Vandenbroucke, G. (2019): "The Baby Boomers and the Productivity Slowdown," Federal Reserve Bank of St. Louis Working Papers. 1, 4

Appendix

A KLEMS and WIOT sectors

This table shows the sector definition in KLEMS and WIOT. The sectors marked red are included in our empirical analysis. For example, we include sectors 50, 51, and 52 rather than the broad sector G "wholesale and retail trade."

Table A.1: List of sectors in KLEMS and WIOT (2013 version)

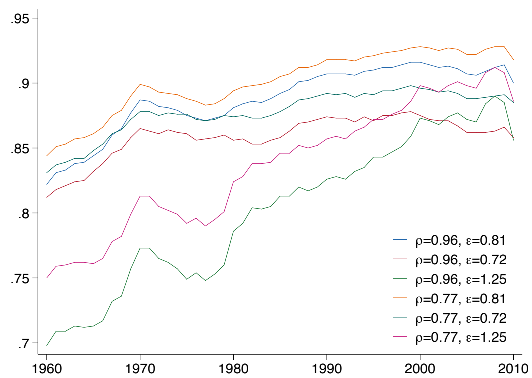
AtB Agriculture hunting forestry and fishing
C Mining and quarrying
D Manufacturing
15t16 Food products, beverages and tobacco
17t19 Textiles, textile products leather and footwear
20 Wood and products of wood and cork
21t22 Pulp paper, paper products, printing and publishing
23 Coke refined petroleum products and nuclear fuel
24 Chemicals and chemical products
25 Rubber and plastics products
26 Other non-metallic mineral products
27t28 Basic metals and fabricated metal products
29 Machinery nec
30t33 Electrical and optical equipment
34t35 Transport equipment
36t37 Manufacturing nec; recycling
E Electricity gas and water supply
F Construction
G Wholesale and retail trade
50 Wholesale trade and commission trade except of motor vehicles and motorcycles
51 Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of fuel
52 Retail trade except of motor vehicles and motorcycles; repair of household goods
H Hotels and restaurants
I Transport and storage and communication
60t63 Transport and storage
64 Post and telecommunications
J Financial intermediation
K Real estate, renting and business activities
70 Real estate activities
71t74 Renting of m&eq and other business activities
M Education
N Health and social work

B Additional results

B.1 CES with different values of elasticity

Figure B.1 displays the evolution of $\mathbf{E_t}$ under different values of elasticity for ρ, ϵ. The blue line shows the result under our baseline parametrization where $\rho = 0.96, \epsilon = 0.81$. Two patterns emerge from this figure. First of all, in terms of the level of measured allocative efficiency, this value is in general lower with a higher elasticity of substitution. Among the six combinations of the parameter values, the lowest allocative efficiency occurs with $\rho = 0.96, \epsilon = 1.25$. This pattern is consistent with findings in Epifani and Gancia (2011) and Osotimehin and Popov (2020). Further, the percent changes in allocative efficiency are also larger for the high-elasticity cases. Secondly, for all six cases, there exist significant deterioration in allocation during the 1970s and either stagnation or deterioration in allocation during the 2000s. In table 6, we formally evaluate the slowdown in productivity under these different parameterizations.

Figure B.1: Evolution of $\mathbf{E}_t$ over time under different values of elasticity



Notes: This figure shows the evolution of allocative efficiency under different values of elasticity of substitution. The baseline parametrization is $\rho = 0.96, \epsilon = 0.81$, which we estimated directly from the data. We also consider a lower value of $\rho = 0.77$ (Aum et al., 2018). For the value of ϵ, we consider two alternative estimates 1.25 (Karabarbounis and Neiman, 2014) and 0.72 (Oberfield and Raval, 2021).

B.2 Alternative measure for capital and labor

Before discussing the results, we first list the asset types provided in the 2009 version of KLEMS. More details can be found in Jorgenson et al. (2014). In this exercise, we consider the most detailed asset classification (eight types) and a more broader classification (two types, ICT versus Non-ICT).

- ICT assets
  - Computing equipment
  - Communications equipment
  - Software
- Non-ICT assets
  - Transport Equipment
  - Other Machinery and Equipment
  - Total Non-residential investment
  - Residential structures
  - Other assets

We first extend our theoretical framework to include more than one asset type. More formally, the planner's optimization problem can be written as

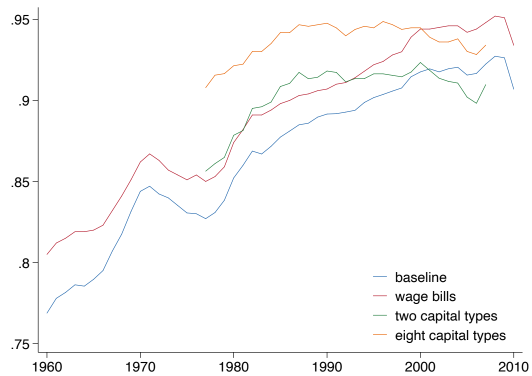
$$\max\ Y_t = \prod_{i=1}^{N} Y_{i,t}^{\theta_{i,t}}, s.t.\ Y_{i,t} = A_{i,t} \prod_{s=1}^{S} K_{i,t}^{s\alpha_{i,t}^s} L_{i,t}^{1-\sum_s \alpha_{i,t}^s},\ \forall s,\ \sum_i K_{i,t}^s = K_t^s,\ \sum_i L_{i,t} = L_t,$$

where $s \in \{1, ..., S\}$ represents the different asset types. The optimal allocation of capital and labor is such that $K_{i,t}^{s*} = \chi_{i,t}^{ks*} K_t^s$, and $L_{i,t}^* = \chi_{i,t}^{l*} L_t$, where $\chi_{i,t}^{ks*} = \frac{\theta_{i,t}\alpha_{i,t}^s}{\sum_i \theta_{i,t}\alpha_{i,t}^s}$ and $\chi_{i,t}^{l*} = \frac{\theta_{i,t}(1-\sum_s \alpha_{i,t}^s)}{\sum_i \theta_{i,t}(1-\sum_s \alpha_{i,t}^s)}$. Lastly, the sufficient statistic for allocative efficiency can be written as $\mathbf{E}_t = \prod_{i=1}^{N}\{\prod_s [(\frac{\chi_{i,t}^{ks}}{\chi_{i,t}^{ks*}})^{\alpha_{i,t}^s}](\frac{\chi_{i,t}^{l}}{\chi_{i,t}^{l*}})^{1-\sum_s \alpha_{i,t}^s}\}^{\theta_{i,t}}$.

Figure B.2 displays the evolution of allocative efficiency under the alternative measures of the capital and labor inputs. The blue line is the benchmark result where we consider only one type of capital and measure labor inputs using employment. Replacing employment with wage bills as measures for labor input, allocative efficiency is slightly higher, but the trend remains very similar to the benchmark result. When we consider two types of assets—ICT, and non-ICT—the time series only start from the end of the 1970s. The allocative efficiency increases rapidly during the 80s, stays relatively stable during the 90s, and starts to decline since the beginning of the 2000s. A similar trend is found when considering eight asset types instead of two, although the level of allocative efficiency is now slightly higher than the two-asset case.

Table B.2 shows the growth rates by decades under optimal allocation. The results with wage bills show that the productivity slowdown completely disappears in the 2000s and it shrinks by half (from 12 pp in the data to 6 pp under the optimal allocation) during the 1970s. Since the capital by type data is only available after 1977, we can only speak about the later slowdown episode. We find that under the optimal allocation, productivity declines by 1 pp and 2 pp for the two-capital case and the eight capital case, respectively. In sum, results in this table indicate that even under these alternative measures, allocation can still explain at least half of the slowdown in the 1970s and one-third of the slowdown in the 2000s.

Figure B.2: Evolution of $\mathbf{E}_t$ over time, alternative measures of inputs



Notes: This figure shows the evolution of allocative efficiency under different measures of capital and labor inputs. The baseline results is the one with one type of capital and where labor inputs is number the employment. The other three alternative measures are 1) using wage bills (labor compensation) instead to measure labor inputs, 2) consider two types of capital (ICT and non ICT), and 3) eight different types of assets.

Table B.2: Productivity growth under optimal allocation, different measures for inputs

	two K types		eight K types		wage bills	
	dy/y	$\Delta dy/y$	dy/y	$\Delta dy/y$	dy/y	$\Delta dy/y$
1960-69					0.19	
1970-79					0.13	-0.06
1980-89	0.11		0.10		0.11	-0.02
1990-99	0.19	0.08	0.19	0.09	0.16	0.05
2000-07	0.18	-0.01	0.17	-0.02	0.16	0.00

Notes: This table presents the productivity growth rates under the optimal allocation (dy/y) and the changes in these growth rates from the previous decade ($\Delta dy/y$). The baseline result is the one with one type of capital and where labor inputs are measured using employment. The other three alternative measures are 1) using wage bills (labor compensation) instead to measure labor inputs, 2) consider two types of capital separately (ICT and non ICT), and 3) eight different types of assets separately.

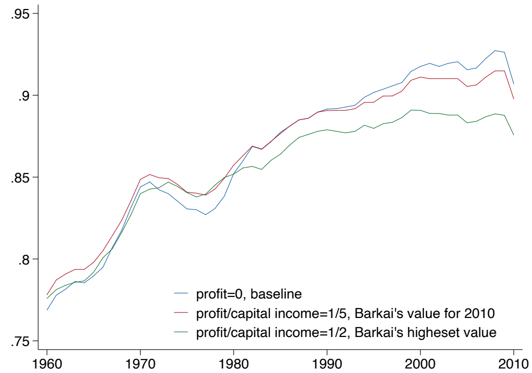
B.3 Non-zero profits

For the profit versus capital income split, we follow Barkai (2020), in which the author estimated the profits and capital income between 1984 and 2014. The results showed that profits as a share of value-added started to increase since the beginning of the 2000s. Over his whole sample period, the ratio of profit to capital income reached the highest level of 1/2 in 2007. In 2010, the last year of our sample, the ratio of profit to capital income was approximately 1/5. We repeat our exercises with these two alternative values. More formally, we reestimate the output elasticity in the production functions after taking out profits from capital returns. As a result, this adjustment lowers the output elasticity for capital and increases that for labor.

Figure B.3 shows the changes in measured allocative efficiency under these two alternative specifications. Compared to the baseline results where we assume zero profits, the magnitude of the changes in allocation is generally smaller. However, for the periods of interest, there still exists an apparent stagnation or deterioration in allocation.

In table B.3, we evaluate more formally the role of allocation under assumptions of positive profits. With a profit to capital income ratio of 1/5, the productivity slowdown under the optimal allocation is 5pp and 1pp during the 1970s and 2000s, slightly higher than the baseline model of 4 pp and 1 pp, yet still significantly smaller than the 12 pp and 3 pp in the data. If the ratio of profit to capital income increases to 1/2, the productivity slowdown becomes only slightly more severe at 6 pp and 1 pp for these two periods. In sum, our main conclusion stands under these alternative assumptions of positive profits. We find consistently that at least half of the slowdown in the data can be attributed to allocation.

Figure B.3: Evolution of $\mathbf{E}_t$ over time, non-zero profits



Notes: This figure shows the evolution of allocative efficiency under different profit to capital income ratios. The baseline results is the one with zero profits. The two other values are taken from Barkai (2020): (i) 1/5, for 2010, (ii) 1/2 the highest point for the period 1984-2014.

Table B.3: Productivity growth under optimal allocation, non-zero profits

	$\frac{profit}{capital\ income} = \frac{1}{5}$		$\frac{profit}{capital\ income} = \frac{1}{2}$	
	dy/y	$\Delta dy/y$	dy/y	$\Delta dy/y$
1960-69	0.17		0.18	
1970-79	0.13	-0.05	0.11	-0.06
1980-89	0.11	-0.02	0.12	0.00
1990-99	0.17	0.06	0.18	0.06
2000-07	0.16	-0.01	0.17	-0.01

Notes: This table presents the productivity growth rates under the optimal allocation (dy/y) and the changes in these growth rates from the previous decade ($\Delta dy/y$). In the figure we plot three specifications with different profit to capital income ratios. The baseline results is the one with zero profits. The two other values are taken from Barkai (2020): (i) 1/5, for 2010, (ii) 1/2, the highest value during his sample period.

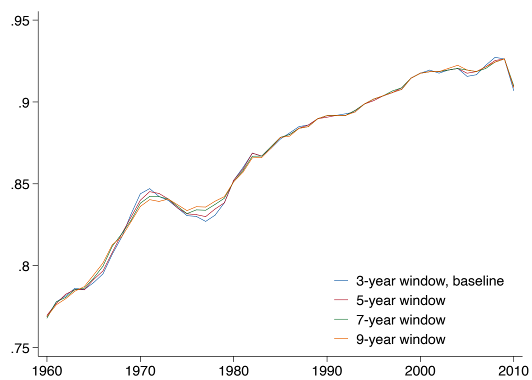
B.4 Length of the rolling window

Our baseline specification relies on the assumption that, although the expenditure shares of the inputs might be distorted each year, they are undistorted on average over time. Consequently, the output elasticities in the production system are equal to the average inputs expenditure shares. In practice, we consider a rolling window of 3 years centered in the current period—[t-1,t,t+1] for period t—and assign the average expenditure shares within this rolling window to the output elasticity in year t.

Next, we examine how sensitive the analysis is to the length of the rolling windows. In figure B.4, we show the evolution of allocative efficiency using a rolling window of 3, 5, 7, and 9 years. The results suggest that a longer rolling window tends to make the lines smoother, however, it does not seem to alter the magnitude of changes significantly.

This finding is also confirmed by table B.4, where we present the changes in growth rates under the optimal allocation. Compared to the baseline setup of a 3-year window, the productivity slowdown is slightly more prominent under the setup of a longer window. With a rolling window of 9 years, the productivity slowdown was 5 pp in the 1970s and 1pp in the 2000s. We find that the length of the rolling window does not have a significant impact on our quantitative results.

Figure B.4: Evolution of $\mathbf{E}_t$ over time, alternative rolling windows



Notes: This figure shows the evolution of allocative efficiency. The baseline results is the one with zero profits with different lengths of the rolling windows.

Table B.4: Productivity growth under optimal allocation, alternative rolling windows

	5 year		7 year		9 year	
	dy/y	$\Delta dy/y$	dy/y	$\Delta dy/y$	dy/y	$\Delta dy/y$
1960-69	0.17		0.17		0.17	
1970-79	0.13	-0.04	0.12	-0.04	0.12	-0.05
1980-89	0.10	-0.03	0.10	-0.02	0.10	-0.02
1990-99	0.16	0.06	0.17	0.06	0.17	0.06
2000-07	0.16	-0.01	0.16	-0.01	0.16	-0.01

Notes: This table presents the productivity growth rates under the optimal allocation (dy/y) and the changes in these growth rates from the previous decade ($\Delta dy/y$) under different lengths of the rolling windows

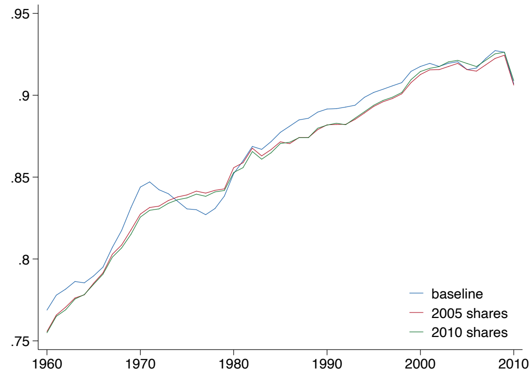
B.5 Using expenditure shares of later years

In this section, we consider one more specification. The assumption behind this specification is that resources are more efficiently allocated in the later years than the earlier years; therefore the expenditure shares of the later years are more likely to be undistorted and equal to the real output elasticity in the production functions. Therefore, the output elasticity can be backed out using the expenditure shares of the later years.

We first choose 2010 as the base year for this exercise, since it is the last year of the sample. To avoid the impact of the Great Recession, we also pick 2005 as an alternative.[28] In figure B.5, the long-run changes in the allocative efficiency under these alternative specifications are rather similar to that under the baseline specification, so is the development during the slowdown episode of the 2000s. During the 1970s, there is a significant slowdown in the improvement of allocation—the line is basically flat during this period, instead of deterioration under the baseline specification. As shown in table B.5, in both exercises, we find a slowdown in productivity of 6 pp during the 1970s and essentially no slowdown in productivity growth during the 2000s.

[28]Since we estimate the output elasticities using a three-year rolling window, 2005 was the last year that was unaffected by the Great Recession which started in 2007.

Figure B.5: Evolution of $\mathbf{E}_t$ over time, using expenditure shares of later years



Notes: This figure shows the evolution of allocative efficiency where we assume that the factor expenditure shares are undistorted in later years. We pick two base years for this exercise: 1) 2010, the last year of the sample and 2) 2005, to avoid the impact of the Great Recession.

Table B.5: Productivity growth under optimal allocation, expenditure shares of later years

	2005 shares		2010 shares	
	dy/y	$\Delta dy/y$	dy/y	$\Delta dy/y$
1960-69	0.16		0.17	
1970-79	0.11	-0.06	0.11	-0.06
1980-89	0.12	0.01	0.11	0.01
1990-99	0.16	0.04	0.16	0.05
2000-07	0.16	0.00	0.16	0.00

Notes: This table presents the productivity growth rates under the optimal allocation (dy/y) and the changes in these growth rates from the previous decade ($\Delta dy/y$) where we assume that the factor expenditure shares are undistorted in later years. We pick two base years for this exercise: 1) 2010, the last year of the sample and 2) 2005, to avoid the impact of the Great Recession.

C Model details for the value-added economy

Solving planner's problem The solution to the planner's problem requires the equalization of MPK and MPL across sectors, such that,

$$\frac{\partial \log Y}{\partial K_i} = \lambda$$
$$\frac{\partial \log Y}{\partial L_i} = \eta.$$

They can be written as,

$$K_i^* = \frac{\theta_i \alpha_i}{\lambda}$$
$$L_i^* = \frac{\theta_i (1-\alpha_i)}{\eta}$$

Given the resource constraint, we get

$$K_i^* = \chi_{i,t}^{k*} K$$
$$L_i^* = \chi_{i,t}^{l*} L,$$

where $\chi_{i,t}^{k*} = \frac{\theta_i \alpha_i}{\sum_i \theta_i \alpha_i}$ and $\chi_{i,t}^{l*} = \frac{\theta_i(1-\alpha_i)}{\sum_i \theta_i(1-\alpha_i)}$. *Q.E.D.*

Deriving allocative efficiency The final good output under optimal allocation can be written as

$$\begin{aligned} Y^* &= \prod_i Y_i^{*\theta_i} \\ &= \prod_i (A_i K_i^{*\alpha_i} L_i^{*1-\alpha_i})^{\theta_i} \\ &= \prod_i (A_i (\chi_i^{k*} K)^{\alpha_i} (\chi_i^{l*} L)^{1-\alpha_i})^{\theta_i}. \end{aligned}$$

Similarly the final output in the data is

$$\begin{aligned} Y &= \prod_i Y_i^{\theta_i} \\ &= \prod_i (A_i K_i^{\alpha_i} L_i^{1-\alpha_i})^{\theta_i} \\ &= \prod_i (A_i (\chi_i^{k} K)^{\alpha_i} (\chi_i^{l} L)^{1-\alpha_i})^{\theta_i}. \end{aligned}$$

As a result,

$$\mathbf{E}_t = \prod_i \left[\left(\frac{\chi_i^{k*}}{\chi_i^k} \right)^{\alpha_i} \left(\frac{\chi_i^{l*}}{\chi_i^l} \right)^{1-\alpha_i} \right]^{\theta_i}.$$

D Model details for the input-output economy

Solving planner's problem The planner's problem is

$$C = \prod_{i=1}^{N} (Q_i - \sum_{j=1}^{N} d_{ji})^{\theta_i} - \sum_i \sum_j \bar{P}_j m_{ij}.$$

The FOCs for K_i, L_i, d_{ij}, m_{ij} are

$$\begin{aligned}
\frac{\partial C}{\partial K_i} &= \theta_i \frac{Y^*}{Y_i^*} \frac{Q_i^*}{K_i^*} \alpha_i (1 - \sigma_i - \lambda_i) \\
\frac{\partial C}{\partial L_i} &= \theta_i \frac{Y^*}{Y_i^*} \frac{Q_i^*}{K_i^*} (1 - \alpha_i)(1 - \sigma_i - \lambda_i) \\
\frac{\partial C}{\partial d_{ij}} &= \theta_i \frac{Y^*}{Y_i^*} [\frac{Q_i^*}{d_{ij}^*} \sigma_{ij} - I_{\{i=j\}}] + \theta_j \frac{Y^*}{Y_j^*} [\frac{Q_j^*}{d_{jj}^*} \sigma_{jj} I_{\{i=j\}} - 1] \\
\frac{\partial C}{\partial m_{ij}} &= \theta_i \frac{Y^*}{Y_i^*} \frac{Q_i^*}{m_{ij}^*} \lambda_{ij} - \bar{P}_j
\end{aligned}$$

The FOC $\frac{\partial C}{\partial d_{ij}} = 0$ implies

$$d_{ij}^* = \frac{\theta_i Y_j^*}{\theta_j Y_i^*} \sigma_{ij} Q_i^*, \tag{9}$$

therefore

$$\begin{aligned}
Y_j^* = Q_j^* - \sum_{i=1}^{N} d_{ij}^* &= Q_j^* - \sum_{i=1}^{N} \frac{\theta_i Y_j^*}{\theta_j Y_i^*} \sigma_{ij} Q_i^*, \\
Y_j^* [1 + \frac{1}{\theta_j} \sum_i (\frac{\theta_i Q_i^*}{Y_i^*} \sigma_{ij})] &= Q_j^*.
\end{aligned}$$

Letting $\chi_j^{y*} = \frac{Y_i^*}{Q_i^*}$, $\{\chi_i^{y*}\}_{i=1}^N$ solve the following equations

$$\frac{1}{\chi_i^{y*}} = 1 + \frac{1}{\theta_i} \sum_s (\frac{\theta_s}{\chi_s^{*y}} \sigma_{si}) \tag{10}$$

or

$$1 - \chi_j^{y*} = \sum_i \sigma_{ij} \frac{\theta_i \chi_j^{y*}}{\theta_j \chi_i^{y*}}.$$

Letting $\gamma_{ij}^* = \frac{\theta_i \chi_j^{y*}}{\theta_j \chi_i^{y*}} \sigma_{ij}$ in equation 9, then $d_{ij}^* = \gamma_{ij}^* Q_j^*$. The market clearing condition for Q_i^* implies

$$\chi_i^{y*} = 1 - \sum_s \gamma_{si}^*.$$

FOC $\frac{\partial C}{\partial m_{ij}} = 0$ implies

$$m_{ij}^* = \theta_i \frac{Y^*}{Y_i^*} Q_i^* \frac{\lambda_{ij}}{\bar{P}_j} \tag{11}$$

Since

$$Y^* = \prod_i Y_i^{*\theta_i} = \prod_i (\chi_i^* Q_i^*)^{\theta_i}$$

we have

$$m_{ij}^* = \theta_i \prod_s (\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s} \prod_s (Q_s^*)^{\theta_s} \frac{\lambda_{ij}}{\bar{P}_j} \tag{12}$$

The FOC $\frac{\partial C}{\partial K_i}=0$ and $\frac{\partial C}{\partial L_i}=0$ lead to

$$K_i^* = \chi_i^{k*} K \tag{13}$$

$$L_i^* = \chi_i^{l*} L \tag{14}$$

where

$$\chi_i^{k*} = \frac{\frac{\theta_i\alpha_i(1-\sigma_i-\lambda_i)}{(1-\sum_j \gamma_{ji}^*)}}{\sum_s \frac{\theta_s\alpha_s(1-\sigma_s-\lambda_s)}{(1-\sum_j \gamma_{js}^*)}}, \chi_i^{l*} = \frac{\frac{\theta_i(1-\alpha_i)(1-\sigma_i-\lambda_i)}{(1-\sum_j \gamma_{ji}^*)}}{\sum_s \frac{\theta_s(1-\alpha_s)(1-\sigma_s-\lambda_s)}{(1-\sum_j \gamma_{js}^*)}}. \tag{15}$$

To fully characterize d_{ij} and m_{ij},we need to solve for Q_i. Replace d_{ij} and m_{ij} in the production function using $d_{ij}^* = \gamma_{ij}^* Q_j^*$ and 12, we get

$$\begin{aligned} Q_i^* &= A_i(K_i^{*\alpha_i} L_i^{*1-\alpha_i})^{1-\sigma_i-\lambda_i}(\gamma_{i1}^* Q_1^*)^{\sigma_{i1}}\cdots(\gamma_{iN}^* Q_N^*)^{\sigma_{iN}} \prod_{j=1}^N \{\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s} \prod_s (Q_s^*)^{\theta_s} \frac{\lambda_{ij}}{\bar{P}_j}\}^{\lambda_{ij}} \quad (16) \\ &= A_i(K_i^{*\alpha_i} L_i^{*1-\alpha_i})^{1-\sigma_i-\lambda_i}(\prod_{j=1}^N \gamma_{ij}^{\sigma_{ij}})(\prod_{j=1}^N Q_j^{*\sigma_{ij}})[\prod_s (Q_s^*)^{\theta_s}]^{\lambda_i}[\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i} \prod_{j=1}^N (\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}} \\ &= A_i[(\chi_i^{k*}K)^{\alpha_i}(\chi_i^{l*}L)^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^N \gamma_{ij}^{\sigma_{ij}})[\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i} \prod_{j=1}^N (\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}} (\prod_{s=1}^N Q_s^{*\sigma_{is}+\lambda_i\theta_s}). \end{aligned}$$

Define

$$\chi_{Qi}^* = A_i[(\chi_i^{k*}K)^{\alpha_i}(\chi_i^{l*}L)^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^N \gamma_{ij}^{\sigma_{ij}})[\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i} \prod_{j=1}^N (\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}} \tag{17}$$

The above equation can be written as

$$Q_i^* = \chi_{Qi}^* (\prod_{s=1}^N Q_s^{*\sigma_{is}+\lambda_i\theta_s}). \tag{18}$$

Q.E.D.

Deriving allocative efficiency Taking log of equation 18 gives $\log Q_i^* = \log\chi_{Qi}^* + \sum_{j=1}^N [(\sigma_{ij} + \lambda_i\theta_j)\log(Q_j^*)]$. Let $q^* = [\log(Q_1^*), \ldots, \log(Q_N^*)]'_{N\times 1}$, equation 18 can be written as

$$q_{N\times 1}^* = b_{N\times 1}^* + \Omega_{N\times N} q_{N\times 1}^*,$$

where $b^*(i) = \log\chi_{Qi}^*$ and $\Omega(i,j) = \sigma_{ij} + \lambda_i\theta_j$. Therefore q can be solved as $q = Cb^*$ where $C_{N\times N} = (I-\Omega)^{-1}$ and $Q_n^* = \prod_{i=1}^N (\chi_{Qi}^{*C_{ni}})$.

Rewrite equation 17 as

$$\chi_{Qi}^* = z_i^* K^{\alpha_i(1-\sigma_i-\lambda_i)} L^{(1-\alpha_i)(1-\sigma_i-\lambda_i)}$$

where $z_i^* = A_i[(\chi_i^{k*})^{\alpha_i}(\chi_i^{l*})^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^N \gamma_{ij}^{\sigma_{ij}})[\theta_i \prod_s (\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i} \prod_{j=1}^N (\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}$.

Then Q_n^* can be rewritten as

$$Q_n^* = \prod_{i=1}^N (\chi_{Qi}^{*C_{ni}}) = \tilde{A}_n^* K^{\tilde{\alpha}_n} L^{\tilde{\beta}_n} \tag{19}$$

where $\tilde{A}_n^* = \{\prod_{i=1}^N z_i^{*C_{ni}}\}$, and $\tilde{\alpha}_n = \sum_i(\alpha_i(1-\sigma_i-\lambda_i)C_{ni})$, $\tilde{\beta}_n = \sum_i((1-\alpha_i)(1-\sigma_i-\lambda_i)C_{ni})$.

Aggregate output under optimal allocation can be written as a function of aggregate capital K and aggregate labor L

$$Y^* = \prod_i Y_i^{*\theta_i} = \prod_i (\chi_i^{y*}\tilde{A}_i^* K^{\tilde{\alpha}_i} L^{\tilde{\beta}_i})^{\theta_i} = \bar{A}^* K^{\bar{\alpha}} L^{\bar{\beta}}, \tag{20}$$

where $\bar{A}^* = \prod_{i=1}^N (\chi_i \tilde{A}_i^*)^{\theta_i}$ is the aggregate TFP under optimal allocation, and $\bar{\alpha} = \sum_n(\tilde{\alpha}_n\theta_n)$, $\bar{\beta} =$

$\sum_n(\tilde{\beta}_n\theta_n)$.

Replacing Q_s^* in equation 12 using the expression in 19, we can write the expenditure on imported good j as

$$\bar{P}_j m_{ij}^* = [\prod_s(\chi_s^{y*}\tilde{A}_s^*)^{\theta_s}]\{\frac{\theta_i}{\chi_i^{y*}}K^{\sum_s\theta_s\tilde{\alpha}_s}L^{\sum_s\theta_s\tilde{\beta}_s}\}\lambda_{ij} = (\frac{\theta_i\lambda_{ij}}{\chi_i^{y*}})Y^*.$$

The total expenditure on imported goods is

$$X^* = [\sum_{i=1}^N(\frac{\theta_i\lambda_i}{\chi_i^{y*}})]Y^*.$$

The output net of imported goods is

$$C^* = Y^* - X^* = Y^*[1-\sum_{i=1}^N(\frac{\theta_i\lambda_i}{\chi_i^{y*}})].$$

Next, we write the data output Y as a function of data allocation (without the stars). The data analog of equation 16 is

$$Q_i = A_i(K_i^{\alpha_i}L_i^{1-\alpha_i})^{1-\sigma_i-\lambda_i}(\gamma_{i1}Q_1)^{\sigma_{i1}}\cdots(\gamma_{iN}Q_N)^{\sigma_{iN}}\prod_{j=1}^N\{\theta_i\prod_s(\frac{\chi_s}{\chi_i})^{\theta_s}\prod_s(Q_s)^{\theta_s}\frac{\lambda_{ij}}{\bar{P}_j}\}^{\lambda_{ij}}.$$

Let $\chi_{Qi} = A_i[(\chi_i^k K)^{\alpha_i}(\chi_i^l L)^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}(\prod_{j=1}^N\gamma_{ij}^{\sigma_{ij}})[\theta_i\prod_s(\frac{\chi_s}{\chi_i})^{\theta_s}]^{\lambda_i}\prod_{j=1}^N(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}$. The above equation can be written as

$$Q_i = \chi_{Qi}(\prod_{s=1}^N Q_s^{\sigma_{is}+\lambda_i\theta_s}).$$

Let $q = [\log(Q_1),\ldots,\log(Q_N)]'_{N\times1}$, we can solve q as

$$q_{N\times1} = b_{N\times1} + \Omega_{N\times N}q_{N\times1},$$

where $b(i) = \log\chi_{Qi}$ and $\Omega(i,j) = \sigma_{ij}+\lambda_i\theta_j$. Therefore q can be solved as $q = Cb$ where $C_{N\times N} = (I-\Omega)^{-1}$. Therefore,

$$Q_n = \prod_{i=1}^N(\chi_{Qi}^{C_{ni}}) = \tilde{A}_n K^{\tilde{\alpha}_n}L^{\tilde{\beta}_n}.$$

where $\tilde{A}_n = \{\prod_{i=1}^N z_i{}^{C_{ni}}\}$, and $\tilde{\alpha}_n = \sum_i(\alpha_i(1-\sigma_i-\lambda_i)C_{ni})$, $\tilde{\beta}_n = \sum_i((1-\alpha_i)(1-\sigma_i-\lambda_i)C_{ni})$. We can write the data output as

$$Y = \prod_i Y_i^{\theta_i} = \prod_i(\chi_i^y\tilde{A}_iK^{\tilde{\alpha}_i}L^{\tilde{\beta}_i})^{\theta_i} = \bar{A}K^{\bar{\alpha}}L^{\bar{\beta}}, \quad (21)$$

where $\bar{A} = \prod_{i=1}^N(\chi_i^y\tilde{A}_i)^{\theta_i}$ is the aggregate TFP in the data.

In addition, we assume that the expenditure shares of imported intermediate goods are not distorted, such that

$$\bar{P}_j m_{ij} = \lambda_{ij}P_iQ_i = \frac{\lambda_{ij}P_iY_i}{\chi_i^y} = \frac{\theta_i\lambda_{ij}}{\chi_i^y}Y.$$

Thus

$$X = [\sum_{i=1}^N(\frac{\theta_i\lambda_i}{\chi_i^y})]Y$$

and

$$C = Y - X = (1-\sum_{i=1}^N(\frac{\theta_i\lambda_i}{\chi_i^y}))Y.$$

Now we can compute the allocative efficiency as

$$\mathbf{E} = \frac{C}{C^*} = \frac{[1-\sum_{n=1}^{N}(\frac{\theta_n\lambda_n}{\chi_n^y})]\prod_{n=1}^{N}(\chi_n^y\tilde{A}_n)^{\theta_n}K^{\bar{\alpha}}L^{\bar{\beta}}}{[1-\sum_{n=1}^{N}(\frac{\theta_n\lambda_n}{\chi_n^{y*}})]\prod_{n=1}^{N}(\chi_n^{y*}\tilde{A}_n^*)^{\theta_n}K^{\bar{\alpha}}L^{\bar{\beta}}},$$

where

$$\begin{aligned}\frac{\tilde{A}_n}{\tilde{A}_n^*} &= \prod_{i=1}^{N}\{\frac{A_i(\chi_i^{k\alpha_i}\chi_i^{l\,1-\alpha_i})^{1-\sigma_i-\lambda_i}(\prod_{j=1}^{N}\gamma_{ij}^{\sigma_{ij}})[\theta_i\prod_s(\frac{\chi_s^y}{\chi_i^y})^{\theta_s}]^{\lambda_i}\prod_{j=1}^{N}(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}}{A_i(\chi_i^{k*\alpha_i}\chi_i^{l*1-\alpha_i})^{1-\sigma_i-\lambda_i}(\prod_{j=1}^{N}\gamma_{ij}^{*\sigma_{ij}})[\theta_i\prod_s(\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s}]^{\lambda_i}\prod_{j=1}^{N}(\frac{\lambda_{ij}}{\bar{P}_j})^{\lambda_{ij}}}\}^{C_{ni}} \\ &= \prod_{i=1}^{N}\{[(\frac{\chi_i^k}{\chi_i^{k*}})^{\alpha_i}(\frac{\chi_i^l}{\chi_i^{l*}})^{1-\alpha_i}]^{1-\sigma_i-\lambda_i}\frac{[\prod_s(\frac{\chi_s^y}{\chi_i^y})^{\theta_s}]^{\lambda_i}}{[\prod_s(\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s}]^{\lambda_i}}\prod_{j=1}^{N}(\frac{\gamma_{ij}}{\gamma_{ij}^*})^{\sigma_{ij}}\}^{C_{ni}}.\end{aligned}$$

Rearrange, we get,

$$\mathbf{E} = E^{kl}E^dE^mE^y,$$

where $E^{kl}=\prod_{i=1}^{N}(((\frac{\chi_i^k}{\chi_i^{k*}})^{\alpha_{i,t}}(\frac{\chi_i^l}{\chi_i^{l*}})^{1-\alpha_i})^{1-\sigma_i-\lambda_i})^{\sum_n\theta_nC_{ni}}$, $E^d=\prod_{i=1}^{N}(\prod_{j=1}^{N}(\frac{\gamma_{ij}}{\gamma_{ij}^*})^{\sigma_{ij}})^{\sum_n\theta_nC_{ni}}$, $E^m=\frac{1-\sum_{n=1}^{N}\frac{\theta_n\lambda_n}{\chi_n^y}}{1-\sum_{n=1}^{N}\frac{\theta_n\lambda_n}{\chi_n^{y*}}}$ and $E^y=\prod_{n=1}^{N}(\frac{\chi_n^y}{\chi_n^{y*}})^{\theta_{n,t}}\prod_{i=1}^{N}(\frac{\prod_s(\frac{\chi_s^y}{\chi_i^y})^{\theta_s}}{\prod_s(\frac{\chi_s^{y*}}{\chi_i^{y*}})^{\theta_s}})^{\lambda_i\sum_n(\theta_nC_{ni})}$.

In addition, we can show that the value-added aggregate production function that features a constant returns to scale. That is, $\bar{\alpha}+\bar{\beta}=1$. To show this, we only need to show that $(\tilde{\alpha}_n+\tilde{\beta}_n)=1$. Then it follows that $\bar{\alpha}+\bar{\beta}=\sum_n((\tilde{\alpha}_n+\tilde{\beta}_n)\theta_n)=\sum_n\theta_n=1$.

To show that $\tilde{\alpha}_n+\tilde{\beta}_n=1$, let $B=I-\Omega$, therefore

$$\sum_j B(i,j)=\sum_j(1-(\sigma_i+\lambda_i\theta_j))=1-(\sigma_i+\lambda_i).$$

The first equality is because of the definition of Ω. The second equality holds because $\sum_j\theta_j=1$. Note that

$$\tilde{\alpha}_n+\tilde{\beta}_n=\sum_i(C_{ni}(1-\sigma_i-\lambda_i))=\sum_i\sum_j C(n,i)B(i,j)$$

Since by definition, $BC=CB=I$, $\sum_j\sum_i C(n,i)B(i,j)=1$ holds for any n. Therefore $(\tilde{\alpha}_n+\tilde{\beta}_n)=1$.

E Model details for the CES production system

Solving planner's problem Recall the planner's problem.

$$max\ Y = (\sum_i \omega_i Y_i^{1-\frac{1}{\rho}})^{\frac{\rho}{\rho-1}},$$

$$s.t\quad \sum_i K_i = K, \sum_i L_i = L, Y_i = A_i(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}.$$

FOC wrt K_i is

$$[K_i] : \omega_i Y_i^{*1-\frac{1}{\rho}} = \frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})\eta_K Y^{*-\frac{1}{\rho}},$$

where the multiplier η_k measures the marginal production for capital in each sector.

Summing up the LHS of the above equation over all sectors gives

$$Y^{*\frac{\rho-1}{\rho}} = \sum \omega_i Y_i^{*1-\frac{1}{\rho}} = \sum \frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})\eta_K Y^{*-\frac{1}{\rho}},$$

$$\frac{Y^*}{\sum \frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})} = \eta_K = \omega_i Y^{*\frac{1}{\rho}} Y_i^{*-\frac{1}{\rho}} \frac{Y_i^*}{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}} \nu_i K_i^{*-\frac{1}{\varepsilon}},$$

$$\omega_i (\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} = \frac{\frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})}{\sum_i \frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})},$$

where the first equation on the second line follows directly from the first line, and the second equation of the second line comes from the FOC w.r.t. K_i. Note that the numerator can be written as

$$\frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}) = \frac{K_i^{*\frac{1}{\varepsilon}-1}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})K_i^*$$

$$= \frac{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}}} K_i^* = \frac{K_i^*}{\frac{\nu_i K_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}}.$$

Let $\alpha_i^* = \frac{\nu_i K_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}$, rewrite the above equation as

$$\frac{K_i^{*\frac{1}{\varepsilon}}}{\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}) = \frac{K_i^*}{\alpha_i^*}.$$

We next apply a similar approach to labor,

$$\omega_i (\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} = \frac{\frac{L_i^{*\frac{1}{\varepsilon}}}{1-\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})}{\sum_i \frac{L_i^{*\frac{1}{\varepsilon}}}{1-\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}})}.$$

Again the numerator of the above equation can be written as

$$\frac{L_i^{*\frac{1}{\varepsilon}}}{1-\nu_i}(\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}) = \frac{L_i^*}{\frac{(1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{*1-\frac{1}{\varepsilon}}}} = \frac{L_i^*}{1-\alpha_i}.$$

To summarize, we derive so far three equations for the planner's problem,

$$\alpha_i^* = \frac{\nu_i K_i^{*1-\frac{1}{\varepsilon}}}{\nu_i K_i^{*1-\frac{1}{\varepsilon}} + (1-\nu_i) L_i^{*1-\frac{1}{\varepsilon}}},\ \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} = \frac{\frac{K_i^*}{\alpha_i}}{\sum_i \frac{K_i^*}{\alpha_i}},\ \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} = \frac{\frac{L_i^*}{1-\alpha_i}}{\sum_i \frac{L_i^*}{1-\alpha_i}}.$$

To further simplify the notation, let $\tilde{K}_i^* = K_i^*/\alpha_i^*$ and $\tilde{L}_i^* = L_i^*/(1-\alpha_i^*)$. Using the last two equations from the line above, we get $\frac{\tilde{K}_i^*}{\tilde{L}_i^*} = \frac{\sum_i \tilde{K}_i^*}{\sum_i \tilde{L}_i^*}$. Further, let $\tilde{K}^* = \sum_i \tilde{K}_i^*$ and $\tilde{L}^* = \sum_i \tilde{L}_i^*$. The last two equations from the line above can be written as

$$K_i^* = \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} \tilde{K}^* \alpha_i^*;\ L_i^* = \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} \tilde{L}^*(1-\alpha_i^*).$$

Substitute K_i^* and L_i^* in the sector i's production function,

$$Y_i^* = A_i\{\nu_i[\omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} \tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} \tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \omega_i(\frac{Y_i^*}{Y^*})^{1-\frac{1}{\rho}} A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}.$$

Therefore

$$1 = \omega_i(\frac{Y_i^*}{Y^*})^{-\frac{1}{\rho}} \frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*},$$

$$1 = \omega_i^{1-\rho}(\frac{Y_i^*}{Y^*})^{-\frac{1-\rho}{\rho}} [\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{1-\rho},$$

$$1 = \omega_i^{1-\rho}(\frac{Y_i^*}{Y^*})^{\frac{\rho-1}{\rho}} [\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{1-\rho},$$

$$\omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1} = \omega_i(\frac{Y_i^*}{Y^*})^{\frac{\rho-1}{\rho}} = \frac{K_i^*}{\tilde{K}^*\alpha_i^*},$$

$$K_i^* = \alpha_i^*\tilde{K}^*\omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1}.$$

Similarly, we apply a similar approach to labor,

$$L_i^* = (1-\alpha_i^*)\tilde{L}^*\omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1}.$$

Let $H_i = A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}$, we can rewrite the above equations

$$K_i^* = \alpha_i^*\tilde{K}^*\omega_i^{\rho}[\frac{H_i}{Y^*}]^{\rho-1},\ L_i^* = (1-\alpha_i^*)\tilde{L}^*\omega_i^{\rho}[\frac{H_i}{Y^*}]^{\rho-1}.$$

Take them back to sectoral production functions,

$$Y_i^* = A_i(\nu_i K_i^* + (1-\nu_i)L_i^*)^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= A_i(\nu_i(\alpha_i^*\tilde{K}^*\omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1})^{1-\frac{1}{\varepsilon}}$$

$$+(1-\nu_i)((1-\alpha_i^*)\tilde{L}^*\omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1})^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \omega_i^{\rho}[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1} A_i(\nu_i(\alpha_i^*\tilde{K}^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)((1-\alpha_i^*)\tilde{L}^*\omega_i^{\rho})^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \omega_i^\rho [\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1} A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}$$

$$= \omega_i^\rho Y^{*1-\rho}\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^\rho.$$

Take Y_i^* back to the final good production function,

$$Y^* = (\sum_i \omega_i(\omega_i^\rho Y^{1-\rho}\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^\rho)^{1-\frac{1}{\rho}})^{\frac{\rho}{\rho-1}},$$

$$Y^{*\rho-1} = \sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}.$$

Sum K_i^* and L_i^* up across all sectors,

$$K = \sum_i K_i^* = \sum_i \alpha_i^*\tilde{K}^*\omega_i^\rho[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1},$$

$$L = \sum_i L_i^* = \sum_i (1-\alpha_i^*)\tilde{L}^*\omega_i^\rho[\frac{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}}{Y^*}]^{\rho-1}.$$

Divide both sides by $\tilde{K}^*$ or $\tilde{L}^*$,

$$\frac{K}{\tilde{K}^*} = \frac{\sum_i \alpha_i^*\omega_i^\rho\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}{\sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}},$$

$$\frac{L}{\tilde{L}^*} = \frac{\sum_i (1-\alpha_i^*)\omega_i^\rho\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}{\sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}.$$

Recall that $\frac{K}{\tilde{K}^*} + \frac{L}{\tilde{L}^*} = 1$. Let $\alpha^* = \frac{K}{\tilde{K}^*}$. Then it follows that $1 - \alpha^* = \frac{L}{\tilde{L}^*}$. Rewrite the above equations with α^*,

$$\alpha^* = \frac{\sum_i \alpha_i^*\omega_i^\rho\{A_i\{\nu_i[\tilde{K}^*\alpha_i^*]^{1-\frac{1}{\varepsilon}} + (1-\nu_i)[\tilde{L}^*(1-\alpha_i^*)]^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}{\sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}},$$

$$1 = \frac{\sum_i \frac{\alpha_i^*}{\alpha^*}\omega_i^\rho\{A_i\{\nu_i(K\frac{\alpha_i^*}{\alpha^*})^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(L\frac{1-\alpha_i^*}{1-\alpha^*})^{1-\frac{1}{\varepsilon}}\}^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}{\sum_i \omega_i^\rho\{A_i[\nu_i(K\frac{\alpha_i^*}{\alpha^*})^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(L\frac{1-\alpha_i^*}{1-\alpha^*})^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}}.$$

Deriving allocative efficiency We have derived the optimal output as

$$Y^* = \{\sum_i \omega_i^\rho\{A_i[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}\}^{\frac{1}{\rho-1}}.$$

Now replacing $A_i = \frac{Y_i}{(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i)^{\frac{\varepsilon}{\varepsilon-1}}}$ in this equation yields[29]

$$Y^* = \{\sum_i \omega_i^\rho\{\frac{Y_i}{(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}}[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}\}^{\rho-1}\}^{\frac{1}{\rho-1}}$$

$$= \{\sum_i \omega_i^\rho Y_i^{\rho-1}\{\frac{[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}}{(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}}\}^{\rho-1}\}^{\frac{1}{\rho-1}}.$$

[29] Note that here Y_i, K_i, L_i are the data whereas Y_i^*, K_i^*, L_i^* are the optimal allocation derived from the planner's problem.

Substitute $Y_i = (\frac{P_i Y_i/\omega_i}{PY})^{\frac{\rho}{\rho-1}} Y$ in the above equation,

$$Y^* = \{\sum_i \omega_i^\rho (\frac{P_i Y_i/\omega_i}{PY})^\rho Y^{\rho-1} \{\frac{[\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}]^{\frac{\varepsilon}{\varepsilon-1}}}{(\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}}\}^{\rho-1}\}^{\frac{1}{\rho-1}}$$

$$\mathbf{E_t} = \frac{Y^*}{Y} = \{\sum_i (\frac{P_i Y_i}{PY})^\rho \{\frac{\nu_i(\tilde{K}^*\alpha_i^*)^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(\tilde{L}^*(1-\alpha_i^*))^{1-\frac{1}{\varepsilon}}}{\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}}}\}^{\frac{\varepsilon}{\varepsilon-1}(\rho-1)}\}^{\frac{1}{\rho-1}}$$

$$= \{\sum_i (\frac{P_i Y_i}{PY})^\rho \{\frac{\nu_i(K\frac{\alpha_i^*}{\alpha^*})^{1-\frac{1}{\varepsilon}} + (1-\nu_i)(L\frac{1-\alpha_i^*}{1-\alpha^*})^{1-\frac{1}{\varepsilon}}}{\nu_i K_i^{1-\frac{1}{\varepsilon}} + (1-\nu_i)L_i^{1-\frac{1}{\varepsilon}}}\}^{\frac{\varepsilon}{\varepsilon-1}(\rho-1)}\}^{\frac{1}{\rho-1}}.$$

where the last equation is derived by replacing $\tilde{K}^*$ with K/α^*.

F The input-output linkages

Recall that adding input-output linkages alters the measurement of allocation in two ways: 1) it accounts for the allocation of intermediate inputs, and 2) the sectoral weights representing the relative importance of each sector take into account the input-output effects. In this section, we provide more details regarding the second point, i.e., the sectoral weights. More specifically, there are two sets of sectoral weights.

First, focusing on the terms that measure capital and labor allocation, equation 2 and the E_{kl} term in equation 4, we see that both are a weighted geometric average of sectoral level allocative efficiency, but with different sets of sectoral weights: the weights are θ_i in the value-added economy and $(1-\sigma_i-\lambda_i)\sum_n \theta_n C_{ni}$ in the input-output economy, where C is the Leontief inverse matrix.

Second, we use a different set of sectoral-weight to solve for the optimal allocation: in the value added economy, the sectoral weight is again θ_i, and in the input-output economy, the weight is $\theta_{i,t}(1-\sigma_{i,t}-\lambda_{i,t})/(1-\sum_j \gamma^*_{ji,t})$.[30]

A natural question to ask is then, do these weights differ across the two economies? We find that, in an undistorted economy without imported intermediate goods, they are the same and all equal to the sectoral value-added shares.

(i) θ_i in the value-added economy The FOC of the final good producer gives $\theta_i = \frac{P_i Y_i}{Y}$, where $P_i Y_i$ is the value-added output of sector i and Y is the total value-added output (final good price normalized to 1).

(ii) $(1-\sigma_i-\lambda_i)\sum_n \theta_n C_{nj}$ in the input-output economy Note that without international trade, matrix C is just the Leontief inverse matrix. Therefore $\sum_n \theta_n C_{ni}$ is equal to the undistorted Domar weight—sector i's sales (gross output) to GDP. Multiplying it by sector i's value-added share $(1-\sigma_i-\lambda_i)$ yields the value-added share of sector i.

(iii) $\frac{\theta_i(1-\sigma_i-\lambda_i)}{1-\sum_j \gamma^*_{ji}}$ in the input-output economy We only need to show that $\frac{\theta_i}{1-\sum_j \gamma^*_{ji}}$ is equal to the undistorted Domar weight: $\frac{\theta_i}{1-\sum_j \gamma^*_{ji}} = \frac{\theta_i}{\chi^*_i} = \frac{P_i \theta_i Q^*_i}{P_i Y^*_i} = \frac{P_i Q^*_i}{Y^*}$. The first equality is derived from the resource constraint on good i. The second equality holds because of the definition of χ^*_i, such that $\chi^*_i = Y^*_i / Q^*_i$. The last equality holds because of the optimality condition of the final good production: $\theta_i Y^* = P_i Y^*_i$.

What we discussed above is an equivalence result between the value-added economy and the input-output economy. The equivalence result might break down if the economy is distorted or there are imported intermediate goods. We plan to explore this further in future research.

[30] To see this, take capital allocation as an example: the optimal shares are equal to $\frac{\theta_{i,t}\alpha_{i,t}}{\sum_i \theta_{i,t}\alpha_{i,t}}$ and $\frac{\theta_{i,t}\alpha_{i,t}(1-\sigma_{i,t}-\lambda_{i,t})/(1-\sum_j \gamma^*_{ji,t})}{\sum_s[\theta_{s,t}\alpha_{s,t}(1-\sigma_{s,t}-\lambda_{s,t})/(1-\sum_j \gamma^*_{js,t})]}$ in the value-added and input-output economies, respectively. These optimal shares reflect the relative importance of sectors' capital in producing the final good, which are $\theta_{i,t}\alpha_{i,t}$ and $\theta_{i,t}\alpha_{i,t}(1-\sigma_{i,t}-\lambda_{i,t})/(1-\sum_j \gamma^*_{ji,t})$. In other words, since $\alpha_{i,t}$ represents the relative importance of capital in sector i, the sectoral weights are $\theta_{i,t}$ and $\theta_{i,t}(1-\sigma_{i,t}-\lambda_{i,t})/(1-\sum_j \gamma^*_{ji,t})$, respectively.